

Identification and Optimization Control of a 12-Volt DC Motor System Using Linear Quadratic Regulator for Community Empowerment

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Abstract: *Direct current (DC) motors are among the most commonly utilized electric motors in various industries due to their robust and reliable regulatory characteristics. These motors also hold significant potential for application in community-based programs, particularly in renewable energy and small-scale mechanization projects that aim to empower underprivileged communities. To effectively analyze a DC motor system, it is essential to mathematically model its operational variables. This mathematical model is expressed as a transfer function, which is integrated into the simulation process using the Matlab Simulink platform. Typically, first- and second-order equations are used to represent these transfer functions. The optimization process involves the state-space representation to determine the K gain value, which is critical for achieving precise control. The Q value, derived from the multiplication of the C transpose and C matrix, directly influences the system's step response speed, while the R value is predetermined at 0.000001. Adjusting these parameters enables an optimized balance between response speed and system stability. This research provides a foundational framework for leveraging DC motor optimization in real-world applications, particularly in community empowerment programs. By enabling more efficient control mechanisms, this study contributes to the development of affordable and sustainable energy solutions, such as small-scale irrigation systems, local production facilities, or microgrid systems in remote areas.*

Keyword: *transfer function, state space, LQR, LQT*

Introduction

Actuator components, ranging from low-power household electronic devices to high-power industrial applications, widely utilize electric motors. According to the law of energy conservation, electric motors convert electrical energy into mechanical energy [1][5]. Electric motors are classified into two main categories: alternating current (AC) motors and direct current (DC) motors. AC motors operate through alternating current voltage, whereas DC motors rotate due to direct current voltage input [2][6]

In the fields of industry, electronics, and electronic instrumentation, DC motors are in

high demand due to their numerous advantages. These include high torque output, the absence of reactive power losses, and no harmonic distortions in the power supply. Furthermore, DC motors exhibit high control accuracy, making them suitable for precision applications[3][4]. Designing a DC motor system requires an understanding of the system's response, including its reaction to various inputs, stability, and control system performance [7].

Numerous studies have investigated the control of rotational motion. For instance, research by Akbar et al. (2016) employed the Linear Quadratic Regulator (LQR) method to control rotational motion in quadcopters and

Linear Quadratic Tracking (LQT) to manage lateral motion [1]. In this study, the Q and R matrices were optimized using the Genetic Algorithm (GA) method. Similarly, Fahmizal et al. (2018) explored the rotational motion control of DC motors using LQR and compared its performance with conventional Proportional-Integral-Derivative (PID) control. In their study, a black-box identification model was utilized, where the Q matrix elements were derived from the product of the transpose of the C matrix with the C matrix itself, while the R matrix elements were tuned manually [2][5].

This study focuses on the automation control of a DC motor using the LQR and LQT methods. The plant under examination is a 12-volt DC motor branded Geartisan. The input/output system of the Geartisan 12-volt DC motor plant is simulated using the Matlab application. This research aims to analyze the system response for first-order and second-order transfer functions. Unlike previous studies that predominantly addressed modeling and control speed response analysis of DC motors, this research emphasizes achieving a rapid stabilization of the system response following the application of LQR and LQT automation control.

Methodology

The analysis of system modeling involves four key stages of system identification. First, the selection of relevant variables is critical to ensure the model captures the necessary dynamics[6][10]. Second, conceptualization of variable interactions and changes is performed to outline the relationships between these variables. Third, the structural framework of

the model is defined, establishing its mathematical representation. Finally, the general properties of the model, such as stability, are exploited to assess its robustness and applicability.

During this analytical process, if the comparison between the model analysis and available data is unsatisfactory, the model structure must be revised. This involves modifying the framework and gathering additional observational data supported by prior research findings. A thorough parameter estimation process is essential for establishing a reliable model. Once system identification is complete, model validation is required to confirm its accuracy and applicability as a solution framework [11].

One of the most commonly used modeling approaches is mathematical modeling. Mathematical models are idealized representations of real-world systems expressed in the form of symbols and mathematical statements[12]. These models provide quantitative and logical relationships that describe the system in a concise manner. By utilizing mathematical modeling, complex problems can be articulated succinctly, making the overall problem structure more comprehensible. Additionally, mathematical models help uncover critical cause-and-effect relationships within the system.

Simulation and optimization are fundamental approaches used to solve planning and modeling challenges. Simulation serves as a quantitative method for describing system behavior. While simulation cannot directly identify optimal operations or designs, it acts as a tool to evaluate expected outcomes of specific operations and designs. Simulations are used

to estimate the output response of a system based on its inputs. In situations where formulating solutions is challenging, simulations provide a viable method for obtaining relevant and practical answers.

Optimization, in the context of mathematical relationships, refers to the process of maximizing or minimizing a specific objective. Optimization within simulations aims to provide optimal responses to objective functions, types of constraints, and the number of decision variables. This study employs mathematical modeling using simulation and optimization methods. These methods are implemented within the Matlab Simulink application to achieve the desired outcomes.

1. Mathematic model

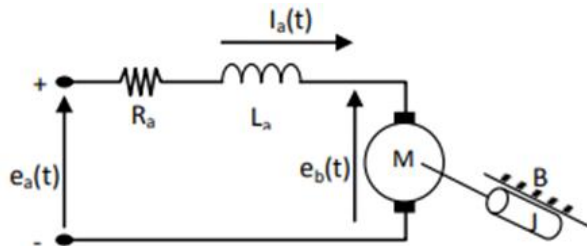


Figure 1. Simple model dc motor

In a simple DC motor model there are parameters including: Force. Force can be caused by electromagnets, gravity, or a combination of other forces[13]. The force has SI newtons (N). Torque, torque is a twisting force that can cause an object to rotate. The torque on the motor shaft is proportional to the current I induced by the input voltage. Inertia, the law of inertia states that an object will tend to remain motionless unless subjected to an external force[14]. Friction force, due to friction which absorbs energy from the

mechanical system, requires a continuous force to keep the object moving. Speed, a moving object will cover a certain distance in a certain time[15][16]. The rotational speed of an object is generally expressed in revolutions per minute (RPM-revolution per minute). An object that manages to make ten complete revolutions in one minute has a speed of 10 RPM [17]. From a simple model of a DC motor, first order and second order transfer function analysis can be obtained.

Below is the specification of dc motor:

Table 1. Specification dc motor

No.	Parameter	Value
1	Voltage	12volt
2	Current	0,6 Amp
3	Torque	1,5 Kg.cm
4	Reduction ratio	31,6
5	No-load speed	150 RPM
6	No-load current	0,15 Amp

• Orde 1

A first order system has a variable s with the highest power being one. The mathematical model of the first order system of a DC motor can be written as follows:

$$\frac{C(s)}{R(s)} = \frac{K}{T + K} \quad (1)$$

$$K = \frac{T}{i}$$

When:

T = torque (Nm)

I = current (A)

- Orde 2

A second order system has a variable s with the highest power being two. The mathematical model of the first order system of a DC motor can be written as follows:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \quad (2)$$

when :

$$\omega_n = \text{no_load_speed}$$

$$\zeta = \text{damping_ratio}$$

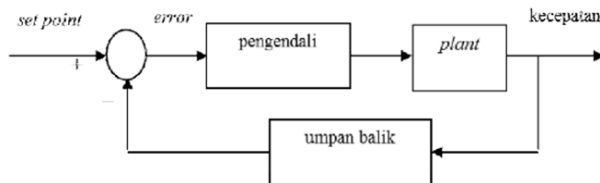


Figure 2 loop diagram

2. Linier Quadratic Regulator

LQR is an optimal control that aims to bring the final state to zero by minimizing the cost function. This type of control aims to find the gain state feedback (K) value [18]. LQR is an optimal control which means the best results that can be achieved by paying attention to the conditions and constraints of a system. Optimal control is generally used to select the input plant u with the minimum performance index [19]. LQR is called linear because the model and shape of the controller are linear. Meanwhile, it is called quadratic because the cost function is quadratic and because the reference is not a function of time, it is called a regulator [20]. A good control system is a control system that has fast and

stable responsiveness, but does not require excessive energy. Such a control system can be achieved through setting appropriate performance indices. A control system designed based on optimizing performance indices is called an optimal control system (Industri, 2016). The general form of the linear system state equation is shown by Equation (3)

$$\begin{aligned} \dot{x} &= Ax(t) - Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (3)$$

when:

$$X_{n+1} = \text{state sistem}$$

$$U_{m \times n} = \text{state input}$$

$$y_{l+1} = \text{state output}$$

$$A = \text{matrik } A$$

$$B = \text{matrik } B$$

$$C = \text{matrik } C$$

The performance index of minimum energy (cost function/quadratic function) in the interval $[t_0 \text{ } t_f]$ is shown by Equation (4).

$$J = \frac{1}{2} \int_0^\infty (x^T Q + u^T R u) dt \quad (4)$$

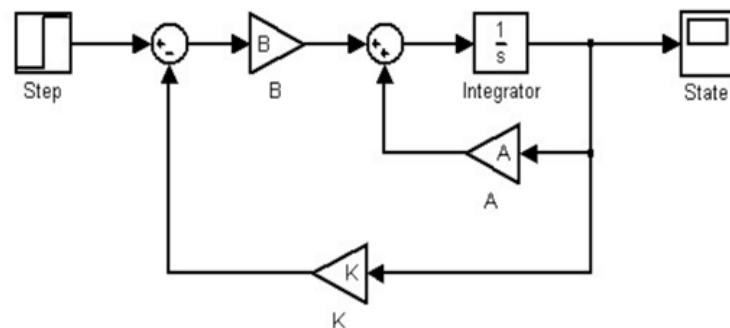


Figure 2 loop diagram LQR

Results and Discussions

1. Orde 1

$$T = 0,147 Nm$$

$$i = 0,6 A$$

$$K = \frac{T}{i}$$

$$K = \frac{0,6}{0,147}$$

$$K = 0,245 Nm / A$$

$$G(s) = \frac{0,245}{0,147s + 0,245}$$

2. Orde 2

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

$$G(s) = \frac{15,7^2}{s^2 + 2.31,6.15,7 + 15,7^2}$$

$$G(s) = \frac{246,49}{s^2 + 492,98 + 246,49}$$

3. LQR

Before creating optimal LQR control, first change the transfer function into state space using programming in Matlab. The conversion results into state space are listed in Table 2.

Table 2. Specification dc motor

Transfer function motor DC	State space motor DC
Orde 1 $G(s) = \frac{0,245}{0,147s + 0,245}$	$A = -1.6667$ $B = 1$ $C = 1.6667$ $D = 0$

Orde 2 $G(s) = \frac{246,49}{s^2 + 492,98 + 246,49}$	$A = \begin{pmatrix} -99.2 & -246.49 \\ 1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $C = (0 \quad 246.49)$ $D = 0$
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nalyze controllability and observability using the commands $\text{rank}(\text{ctrb}(A,B))$ and $\text{rank}(\text{obsv}(A,C))$. The system is said to be stable if the controllability and observability values are more than 0. The controllability and observability results of the DC motor model are shown in Table 3

Table 3. Controllability dan Observability

State space model motor DC	controllability	observability
	1	1
$A = \begin{pmatrix} -99.2 & -246.49 \\ 1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $C = (0 \quad 246.49)$ $D = 0$		2

it is concluded that the system can be controlled and observed. So next look for the elements of Matrix K with LQR. This Matrix K element can be obtained by running where S is the solution to the Riccati equation and E is the following command in Matlab "[K,S,E]=lqr(A,B,Q,R)" closed-loop eigenvalue. From the results of this command, the K element matrix obtained for each model is explained in Table 4. The first experiment is by giving the element matrix values $Q = \text{transpose}(C)*C$ and $R = 0.000001$.

Table 4. K value

State space	K
$A = -1.6667$ $B = 1$ $C = 1.6667$ $D = 0$	1.6650e+03
$A = \begin{pmatrix} -99.2 & -246.49 \\ 1 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $C = (0 \quad 246.49)$ $D = 0$	$1.0e+05 *$ $0.0061 \quad 2.4624$

The following table is a program listing in the Matlab m-file to search for Matrix K elements in the second order model approach using LQR.

```

clc
clear all
num=0.245;
den=[0.147 0.245];
G=tf(num,den)

figure(1)
step(G)

[AX,BX,CX,DX]=tf2ss(num,den)
modelMotor=ss(AX,BX,CX,DX)
modelMotor=tf(modelMotor);

rank(ctrb(AX,BX))
rank(observ(AX,CX))

R=0.000001
Q=transpose(CX)*CX

[K,S,E]=lqr(AX,BX,Q,R)

sys=ss(AX,BX,CX,DX);
Af=AX-BX*K;
T=ss(Af,BX,CX,DX);
T1=tf(T)
figure(2)
step(T1)

```

The response results show that the response step for the first order transfer function with LQR has a better response step. This can be seen in Figure 5. The steady state system takes 6 seconds, while in Figure 6 the system is able to steady state in only 0.006 seconds.

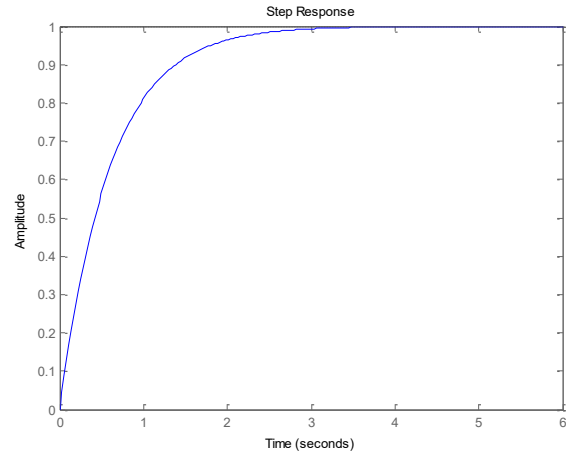


Figure 3. respond ordo 1

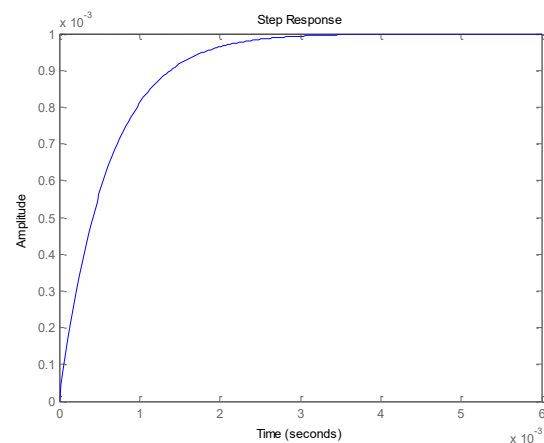


Figure 4. respond ordo 1 with LQR

The following table is a program listing in the Matlab m-file to search for Matrix K elements in the second order model approach using LQR.

Conclusion

The optimal control technique discussed in this paper is LQR, where the Q

element matrix is found by multiplying the transpose of the C matrix of the system with the C matrix of the system.

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