

Comparison of the Use of Linear Quadratic Regulator and Linear Quadratic Tracker Optimal Control Techniques in DC Motor Systems with Noise Addition for the Development of Sustainable Technology Solutions in Community Empowerment

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Abstract: *DC motors are widely used in various sectors, including industrial applications, household appliances, and even children's toys. This research presents a system identification process for a DC motor using experimental techniques with the system identification tool in Matlab. The study explores optimal control techniques, specifically Linear Quadratic Regulator (LQR) and Linear Quadratic Tracker (LQT), to analyze the step response of the system. Understanding the transfer function of the DC motor is crucial for effective control; this is achieved by adjusting the Q and R matrices in the LQR technique, which ultimately modifies the value of the feedback gain, K. The Linear Quadratic Tracking (LQT) system aims to adjust the output to follow a predetermined trajectory, offering a dynamic tracking solution. To apply these control techniques, the DC motor is first modeled in the Laplace domain of order 2 and then transformed into the state-space domain, allowing for integration into LQR and LQT calculations. In this study, noise is introduced to assess the system's performance under both normal and noisy conditions, highlighting the resilience of the control methods. The control process, including the simulation and implementation of LQR and LQT calculations, is conducted using Simulink in Matlab. This research is of particular relevance to community empowerment initiatives, where the principles of optimal control can be applied to improve local technological solutions. The application of these control techniques to affordable, low-cost DC motor systems can significantly contribute to the development of sustainable technology in underprivileged communities, enhancing the accessibility and efficiency of small-scale industrial solutions. The findings aim to support practical applications in fields such as renewable energy, agricultural machinery, and local craftsmanship, offering a pathway to economic growth through technology.*

Keywords: *DC Motor, LQR, LQT, Matlab, Noise, Community Empowerment, Sustainable Technology.*

Introduction

Control systems play a crucial role in engineering, particularly in the fields of industrial applications and everyday life. From household appliances such as washing machines and air conditioners to advanced technologies like robotics in manufacturing and unmanned aerial vehicles (UAVs), control systems ensure the optimal functioning of various devices. The mastery

of control systems is therefore essential, both in theoretical understanding and practical application (Ogata, 2010).

To study and understand the behavior of a system, system identification is required. This involves modeling the system mathematically based on the characteristics of its components. Through system identification, a transfer function is obtained, which allows for an analysis of the

system's response to various inputs (Nise, 2011). With this knowledge, appropriate actions can be taken to ensure that the system behaves as desired. System identification is an experimental approach to determining the dynamic model of a system. Fundamentally, a model is built from observed data. Identifying a system is a crucial step in scientific research, serving as the first phase in any system analysis (Bishop, 2006). However, obtaining a model is not a straightforward task. There are two main approaches for deriving a mathematical model of a physical system: analytical and experimental methods (Simeone, 2018).

Several techniques can be used to derive the model of a DC motor experimentally. One common approach involves observing the input and output data of the motor (Ljung, 1999). Additionally, some researchers employ artificial intelligence algorithms, such as Recurrent Neural Networks (RNN), using artificial neural networks to learn the dynamic model of a DC motor based on known parameters, minimizing the error during the learning process (Hochreiter & Schmidhuber, 1997). Furthermore, research often utilizes feedback voltage from the DC motor as an input variable and motor speed as the output variable. The relationship between input and output is evaluated using Matlab's system identification tool (MathWorks, 2019).

DC motors operate in a nonlinear manner, particularly under varying load conditions. Conventional controllers, such as PID controllers, are widely used due to their good performance in controlling linear systems (Beck, 2019). For example, the induction motor control system has been developed using a PID controller with the

Field Oriented Control (FOC) method, which improves the control signal for torque current in the PID controller system (Huang & Wang, 2016). Additionally, optimal control techniques have been employed by several researchers, including Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT), which provide more precise control in complex systems (Kwakernaak & Sivan, 1972).

The research described in this paper contributes to the empowerment of communities by providing technological solutions that can be implemented in local industries, especially in small-scale manufacturing or renewable energy systems. The application of LQR and LQT methods to control low-cost DC motors presents an opportunity to enhance efficiency and sustainability in community-based technologies (Shao, 2018).

Methodology

1. Linear Quadratic Regulator (LQR)

Linear Quadratic Regulator (LQR) is a widely used method for designing optimal control systems. The design process involves a state-space model that defines the system dynamics. The goal of LQR is to minimize a cost function, which represents the performance of the system, by adjusting the input to drive the output to the desired state. The cost function typically takes the following quadratic form (Anderson & Moore, 2007):

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (1)$$

Where Q is a positive semi-definite matrix, R is a positive definite matrix, x represents the state vector, and u is the control input.

The matrices A and B in the state-space representation are chosen to minimize the above cost function (Bryson & Ho, 1975).

LQR controls a system using a linear combination of the plant's state variables. For effective control, all state variables need to be measurable or estimable. If some state variables are not directly accessible, an observer or estimator can be used to approximate them. The control signal can then be derived from the following equation:

$$u = -Kx \quad (2)$$

Where K is the feedback gain matrix obtained by solving the Riccati algebraic equation. One of the challenges with LQR is the complexity of solving the Riccati equation manually, often requiring computational tools such as MATLAB (Welch & Bishop, 2006). Once the optimal gain matrix K is calculated, it is used to generate the control law for the system.

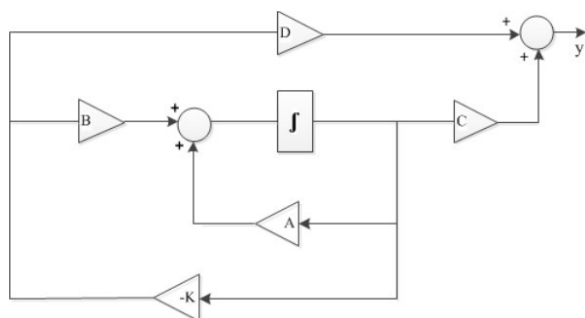


Figure 1. LQR Control System Block Diagram without Input

2. Linear Quadratic Tracking (LQT)

Linear Quadratic Tracking (LQT) is an extension of LQR, where the objective is for the system output to track a desired reference trajectory. The system is modeled with a state-space equation and a tracking

error vector. The performance index for LQT is defined as:

$$J = \int_0^{\infty} (x^T Qx + u^T Ru) dt \quad (3)$$

Where Q and R are matrices that define the performance and the trade-off between the control effort and tracking accuracy. After solving the Riccati differential equation, the feedback gain K is determined (Doyle et al., 1990). LQT can be used in various applications, including robotics and motion control, where precise tracking of a reference trajectory is required (Zhou et al., 1996).

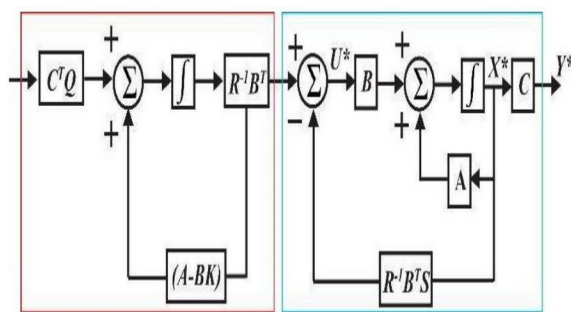


Figure 2. LQT Diagram Block

3. Application to Community Empowerment

In the context of community empowerment, LQR and LQT control systems offer the potential to improve sustainable technologies in small-scale industries. By applying optimal control strategies, such as LQR and LQT, local communities can develop more efficient and reliable systems for agricultural or manufacturing processes. For example, controlling a small-scale water pumping system or renewable energy generation using DC motors can be optimized with these techniques to ensure

higher efficiency and lower operational costs (Cui et al., 2018). The integration of these technologies can support the sustainable development goals by promoting community-based solutions for energy and water access (Maji et al., 2020).

4. Research Stages

This research involves using MATLAB software to model and simulate the performance of DC motors controlled by LQR and LQT systems. The steps include:

- Simulating the systems without noise to observe the basic control performance.
- Introducing noise to evaluate the robustness of the control strategies.
- Comparing the performance of the LQR and LQT systems in terms of stability, tracking accuracy, and disturbance rejection.

The theoretical model for LQR and LQT is derived by first formulating the mathematical state-space models and solving the corresponding Riccati equations. The obtained feedback gains are then applied to simulate the system dynamics.

5. Mathematical Model Calculation for LQR and LQT

The mathematical model for LQR is based on the following parameters for a DC motor:

$$A = \begin{bmatrix} \frac{-R}{L} & \frac{-K}{L} \\ \frac{K}{J} & \frac{-b}{J} \end{bmatrix} \quad (4)$$

$$B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad (5)$$

$$C = [1 \ 0] \quad (6)$$

$$D = 0 \quad (7)$$

Where RRR is the resistance, L is the inductance, K is the motor constant, J is the inertia, and b is the damping coefficient. The control law for LQR is computed by solving the Riccati equation:

$$K = lqr(A, B, Q, R) \quad (8)$$

Similarly, for LQT, the model parameters are defined in a similar manner, with the tracking error introduced as an additional state variable. The optimal control gain for LQT is calculated by solving the associated Riccati differential equation.

6. Simulations without Noise

The performance of both LQR and LQT control systems is simulated using MATLAB without the introduction of noise. The system responses are plotted to analyze how each control method manages the motor speed and position.

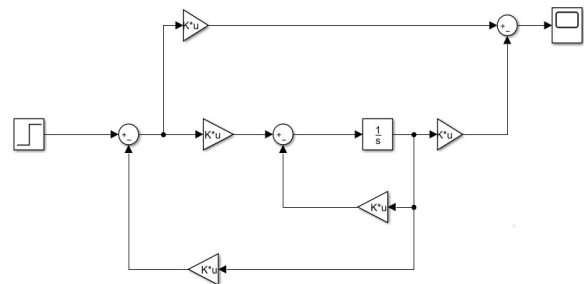


Figure 3. LQR Network Simulation

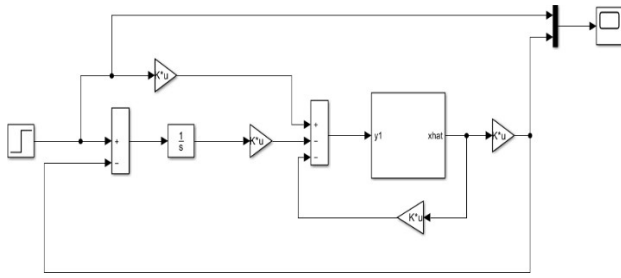


Figure 4. LQT Network Simulation

7. Simulations with Noise

Noise is added to the system to simulate real-world disturbances and test the robustness of the LQR and LQT controllers. The noise is modeled as a Gaussian disturbance, and the control systems' ability to reject this noise is assessed.

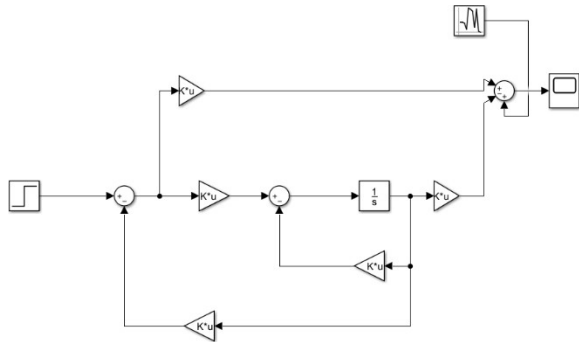


Figure 5. LQR Network Simulation with Noise

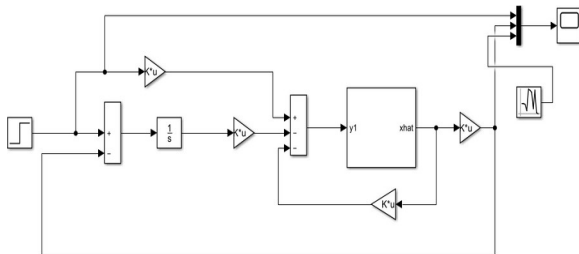


Figure 6. LQT Circuit Simulation with Noise

Regulator (LQR) and Linear Quadratic Tracker (LQT) circuits, both with and without noise.

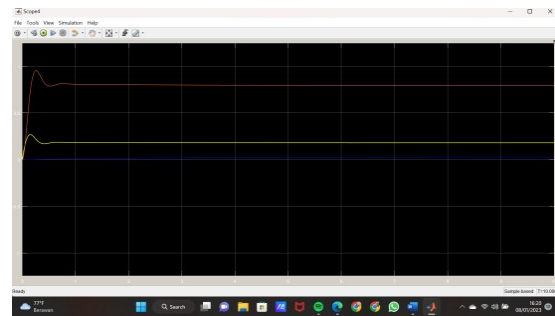


Figure 7. Simulation result of the LQR circuit.

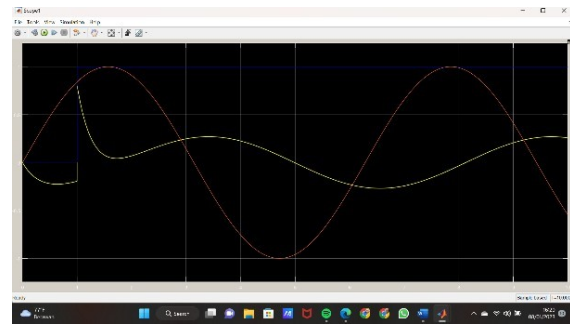


Figure 8. Simulation result of the LQT circuit

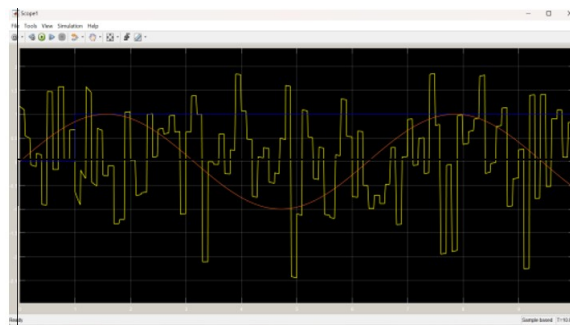


Figure 9. Simulation result of the LQR circuit with noise.

Results and Discussions

1. Simulation Results

Figures 7, 8, 9, and 10 present the simulation results of the Linear Quadratic

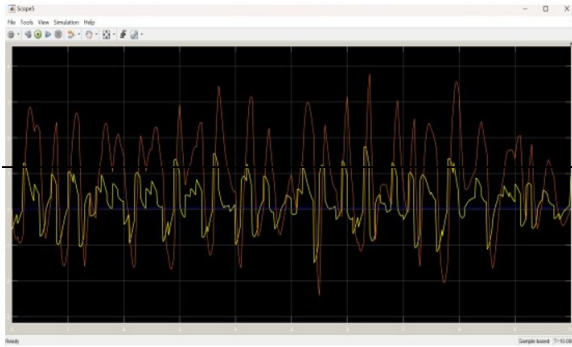


Figure 10. Simulation result of the LQT circuit with noise.

2. Discussion

The conducted tests involved comparing the LQR and LQT circuits using different input types for the plant. The type of input has a significant impact on the output of the system. If the input is not properly matched with the plant's characteristics, the resulting output graphs tend to exhibit considerable ripple effects. In particular, the addition of noise to the system led to outputs with noticeable ripples, indicating instability.

The primary difference between circuits with and without noise lies in the stability of the resulting output. When noise is introduced, the system becomes increasingly inefficient and prone to failure. In practical applications, especially in the context of community service and sustainable technology development, this highlights the importance of designing systems that can tolerate or mitigate the impact of environmental disturbances, such as electrical noise. The ability to maintain system stability under such conditions is crucial for real-world implementations of control systems in fields like agriculture, water treatment, and renewable energy

technologies, where systems may be exposed to noisy environments.

Moreover, the presence of noise significantly disrupts the signal reception process in the system. This failure to accurately process inputs due to external interference affects the system's output, making it less reliable and inconsistent with expected results. These findings are especially relevant when developing solutions for community empowerment and sustainable technology in underserved areas, where robustness and adaptability to varying environmental conditions are key to success.

Conclusion

Based on the conducted experiments, the following conclusions can be drawn:

- a. The implementation of the Linear Quadratic Regulator (LQR) control technique can be effectively adapted for microcontroller-based systems in community service applications, providing an accessible approach for sustainable technological solutions.
- b. The LQR method is capable of controlling the speed of a DC motor, showcasing its potential in motor-driven systems widely used in community development projects.
- c. The system's output aligns with the expected results if the input provided to the plant is well-matched with its requirements, highlighting the importance of accurate system design in community technology solutions.
- d. The introduction of noise in the system results in inefficiency and potential

failure, underlining the need for noise-resistant technologies, particularly in rural or industrial environments that rely on accurate control for sustainable development.

- e. The presence of noise in the system leads to outputs that exhibit significant instability and ripple effects, further emphasizing the importance of noise filtering and robust system design for real-world applications, especially in community-driven technological initiatives.

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