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Utilization of Laplace Transform in Mathematical Modeling of Brushless DC-Servomotors type 1226 012 B and Single-phase AC motors type CSR 90S

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Abstract **The precise control of electric motors, particularly Brushless DC-Servomotors (BLDC) type 1226 012 B and AC single-phase motors type CSR 90S, is central to improving automation systems and industrial applications. However, the inherent complexity of these motors' dynamic behaviors poses significant challenges to accurate mathematical modeling and subsequent control design. This study addresses the problem of developing robust and efficient mathematical models for these motor types, which are critical for system analysis, simulation, and controller development. The principal aim of this research is to utilize Laplace Transform techniques to derive and analyze mathematical models of BLDC and AC single-phase motors, focusing on the dynamic and transient responses under different operational conditions. By formulating the governing differential equations and employing the Laplace Transform, this work streamlines the transition from time-domain analysis to the more tractable frequency-domain approach. The main contribution of this research lies in the development of validated transfer function representations for both the Brushless DC-Servomotors 1226 012 B and the AC single phase Motor type CSR 90S, enabling precise prediction of system responses and performance indices such as rise time, settling time, and steady-state error. In addition, the results will show that the Laplace-based model accurately captures realistic motor behavior, as validated through experimental testing under step input and load disturbance scenarios. The developed model offers reliable prediction capabilities and serves as a solid foundation for advanced control and simulation. In conclusion, the application of Laplace Transform significantly improves the modeling and analysis process for both types of motors, paving the way for optimized control strategies in practical applications.**

Keywords **Laplace Transform; Brushless DC; Single-phase motors; Mathematical models; Transfer function**.

I. Introduction

Mathematical modeling is the main foundation in the design and analysis of electric motor control systems, both AC (alternating current) and DC (direct current) motors. This modeling serves to describe the dynamic behavior of the motor and its control system in a complex manner through mathematical equations in both the time domain and frequency domain. Mathematical modeling in motor control systems is done with transfer functions to predict how a system will react to certain inputs. The purpose of this modeling is to analyze the performance of the motor system, design a control system that can regulate the motor according to the desired goal, predict the system response to input or disturbance and perform simulation before physical implementation. In this study, mathematical modeling was carried out on DC motors and AC single-phase motors using Laplace transform.

Fractional derivative models are particularly useful for precise modeling of systems that require accurate damping representation [1]. Such as laplace transform, one of the benefits of the Laplace transform is that it is used to obtain a mathematical model of a direct current or alternating current electric motor so that a transfer function is obtained which is then used to control the motor, such as motor speed control or using PID [2]. In this research, Brushless DC-Servomotors type 1226

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012 B and single-phase AC motor type CSR 90S are used.

The DC brushless motor is divided into two types because of the different types of its winding drive current waveforms, one is the square wave permanent magnet synchronous Motor, because the armature driver current is the square wave (trapezoidal wave), which is called Brushless DC Motor [3]. A Brushless DC Motor conserves the characteristics of dc motor but eliminates the brushes and commutator hence known as Brushless DC (BLDC) Motor [4].

Compared with other type motor, brushless DC motor in the form of square wave excitation brushless DC motor, improve the utilization rate of the permanent magnetic material and reduced the volume of motor, increasing the motor output, with high efficiency, high reliability characteristics [3].

DC motors usually use direct-unidirectional current so that DC motors can be used to model control systems or others. The importance of modeling DC motor control systems is to understand the behavior of the system and design effective controls. DC motors are widely used in robotics, actuators and precision systems where mathematical models of DC motors allow for highly accurate speed and position control and are used to design closed-loop controls that regulate motors based on feedback from position/speed sensors . DC motors are generally modeled by considering two main aspects, namely electrical equations and mechanical equations.

Meanwhile, Single-phase AC motors are one type of electric motor that is commonly used in household and small-scale industrial applications. In the context of mathematical modeling, this motor can be represented through an analytical approach using the Laplace transform. This transformation allows conversion from the time domain to the frequency domain, so that systems that were originally in the form of differential equations can be simplified into algebraic form. Thus, the dynamic behavior of the motor such as the response of speed, current, and torque to voltage input can be analyzed more systematically [5]. Such modeling generally includes electrical elements in the stator such as resistance and inductance, as well as mechanical elements that affect the dynamics of rotor rotation. One of the advantages of DC motors over AC motors is their easier regulation, especially in terms of speed. In contrast, regulating the speed of AC motors tends to be more complicated because the controls are more complex [6] [7].

Although their control is more complex than DC motors, modeling is important for developing starting systems, speed control, and energy efficiency in AC motors themselves . Single-phase AC motors are often used in cost-effective systems, so modeling can also help design effective systems without expensive hardware. By using modeling, it is possible to predict, design, test, and control AC motor systems with high accuracy [8] [9].

Several recent studies have used differential equation-based modeling methods and domain transformation techniques, particularly the Laplace Transform, to formulate the switching functions of BLDC motors and single-phase AC motors. The Laplace Transform allows complex continuous-time systems to be modeled as transfer functions in the frequent domain, which simplifies stability analysis and control design. Research that integrates mathematical modeling using Laplace Transform specifically still very limited.

To overcome these problems, this study proposes the use of the Laplace Transform method in the preparation of a comprehensive mathematical model for Brushless DC-Servomotors type 1226 012 B and AC single-phase motor type CSR 90S. This method is based on the identification of motor parameters through experiments and the formulation of dynamic differential equations which are then converted to transfer functions. The resulting model will be tested simulatively and experimentally to measure its accuracy and performance.

This study aims to develop and validate a mathematical model based on Laplace transform to represent the dynamic behavior of BLDC motor type 1226 012 B and single-phase AC motor type CSR 90S, and assess the performance of the model in motor simulation and control.

The first contribution of this research lies in the development of transfer function models for the Brushless DC Servomotor type 1226 012 B and the single-phase AC motor type CSR 90S based on Laplace Transform techniques. By experimentally identifying motor parameters and deriving the system's differential equations, this study formulates accurate mathematical representations in the frequency domain. Such analytical models are crucial for precise prediction of dynamic responses and stability analysis, facilitating the design and tuning of advanced control algorithms [10] [11].

Secondly, the research provides model validation through comprehensive simulation and experimental tests. The proposed models' accuracy is verified by comparing simulated motor responses to measured real-time data under various input and load conditions. This step ensures reliability of the model in depicting transient and steady-state behavior, which is essential for practical applications. Experimental validation strengthens the applicability of Laplace-based modeling as shown in recent works that combine

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system identification with simulation tools like MATLAB/Simulink for modeling motor dynamics [10].

The third contribution focuses on establishing a mathematical foundation supportive of effective control system design. By providing clear transfer function formulations, this research equips control engineers with essential tools to implement classical and modern control strategies tailored for these motor types. The precise models reduce uncertainties during controller synthesis, improving responsiveness and reducing steady-state errors, general challenges highlighted in motor control literature [11].

Finally, this research advances the understanding of dynamic characteristics specific to BLDC type1226 012 B and AC CSR 90S motors, which differ from commonly studied generic models. This specificity allows for targeted improvements in efficiency and performance in applications such as robotics or automation systems. By addressing the gap of validated models for these particular motor types, the study contributes to technological development enabling more reliable and optimized motor control solutions in industry [12].

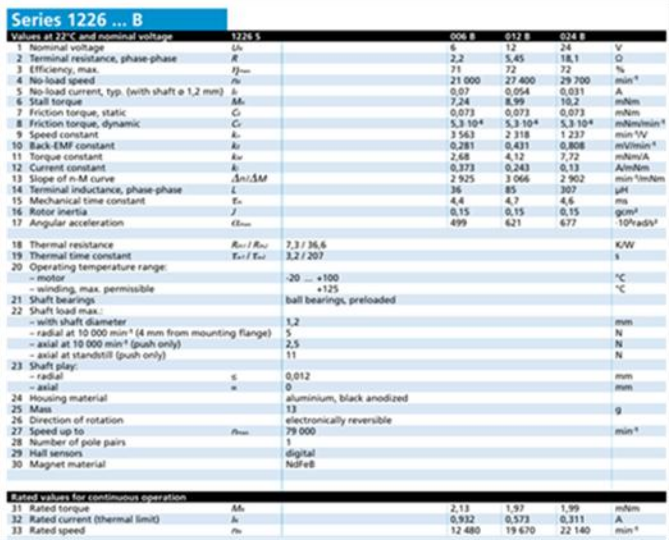
This study is organized as follows: Section II discusses the dataset used, the proposed method in modeling Brushless DC-Servomotors and 1 Phase AC Motor type CSR 90S. Section III displays the results obtained, namely the transfer function. Section IV discusses the interpretation and comparison of the results with other existing research. Section V, the conclusion, restates the objectives, main findings and further research.

II. Method

A. Dataset

The data set used in this study consists of operational parameters collected from Brushless DC-Servomotor type 1226 012 B and AC single phase motor type CSR 90S. These data include several motor parameters thatare needed to be the basis for building an accurate mathematical model so that it can be used to obtain the transfer function and can be further used in the analysis of the motor control system itself.

The first is the datasheet of Brushless DC-Servomotor type 1226 012 B, where the parameter data used includes measurements of resistance, torque constant, back emf, viscous friction, inertia and inductance. The parameters will be used to find a mathematical model of the motor using a transfer function so that the transfer function is obtained.



Series 1226 ... B

	Values at 22°C and nominal voltage	1226 S		006 B	012 B	024 B	
1	Nominal voltage	U_N		6	12	24	V
2	Terminal resistance, phase-phase	R		2,2	5,45	18,1	Ω
3	Efficiency, max.	η_{max}		71	72	72	%
4	No-load speed	n_0		21 000	27 400	29 700	min⁻¹
5	No-load current, typ. (with shaft ø 1,2 mm)	I_0		0,07	0,054	0,031	A
6	Stall torque	M_H		7,24	8,99	10,2	mNm
7	Friction torque, static	C_0		0,073	0,073	0,073	mNm
8	Friction torque, dynamic	C_v		5,3 10⁻⁶	5,3 10⁻⁶	5,3 10⁻⁶	mNm/min⁻¹
9	Speed constant	k_n		3 563	2 318	1 237	min⁻¹/V
10	Back-EMF constant	k_E		0,281	0,431	0,808	mV/min⁻¹
11	Torque constant	k_M		2,68	4,12	7,72	mNm/A
12	Current constant	k_I		0,373	0,243	0,13	A/mNm
13	Slope of n-M curve	$\Delta n/\Delta M$		2 925	3 066	2 902	min⁻¹/mNm
14	Terminal inductance, phase-phase	L		36	85	307	µH
15	Mechanical time constant	τ_m		4,4	4,7	4,6	ms
16	Rotor inertia	J		0,15	0,15	0,15	gcm²
17	Angular acceleration	α_{max}		499	621	677	·10³rad/s²
18	Thermal resistance	R_{th1} / R_{th2}	7,3 / 36,6				K/W
19	Thermal time constant	τ_{w1} / τ_{w2}	3,2 / 207				s
20	Operating temperature range:						
	– motor		-20 ... +100				°C
	– winding, max. permissible		+125				°C
21	Shaft bearings		ball bearings, preloaded				
22	Shaft load max.:						
	– with shaft diameter		1,2				mm
	– radial at 10 000 min⁻¹ (4 mm from mounting flange)		5				N
	– axial at 10 000 min⁻¹ (push only)		2,5				N
	– axial at standstill (push only)		11				N
23	Shaft play:						
	– radial	≤	0,012				mm
	– axial	=	0				mm
24	Housing material		aluminium, black anodized				
25	Mass		13				g
26	Direction of rotation		electronically reversible				
27	Speed up to	n_{max}	79 000				min⁻¹
28	Number of pole pairs		1				
29	Hall sensors		digital				
30	Magnet material		NdFeB				
	Rated values for continuous operation						
31	Rated torque	M_N		2,13	1,97	1,99	mNm
32	Rated current (thermal limit)	I_N		0,932	0,573	0,311	A
33	Rated speed	n_N		12 480	19 670	22 140	min⁻¹

Fig. 1. Datasheet Brushless DC-Servomotors type 1226 012 B

After reviewing the motor datasheet in Fig. 1.,some parameters used for the transfer function were obtained.

Table 1. Parameter of DC Motors

Variable	Unit	Value
Nominal Voltage	V	12
Resistance	Ω	5,45
Current	A	0,054
Inductance	mH	85
Inertia	$Kg.m^2$	$1,5 \times 10^{-8}$
Rated torque	Nm	0,0197
Torque constant	mNm/A	4,12
Back emf	mV sec/rad	0,431
Speed	rpm	2318
Viscous friction	Nms/rad	$9,17 \times 10^{-7}$

And the second is the datasheet of AC single-phase motors type CSR 90S, where the parameter data used includes voltage, speed (rpm), power factor, current and full load torque. The parameters will be used to find a mathematical model of the motor using a transfer function so that the transfer function is obtained.

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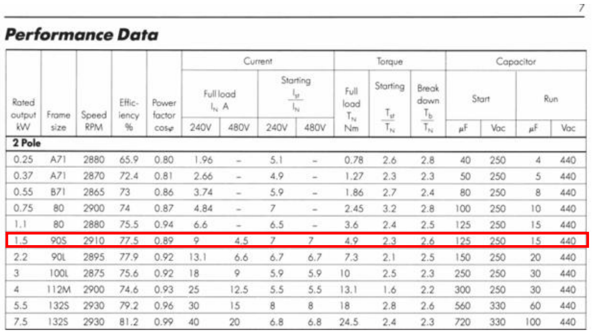

7

Performance Data

Rated output kW	Frame size	Speed RPM	Efficiency %	Power factor cosφ	Current: Full load I_N A 240V	Current: Full load I_N A 480V	Current: Starting $\frac{I_{st}}{I_N}$ 240V	Current: Starting $\frac{I_{st}}{I_N}$ 480V	Torque: Full load T_N Nm	Torque: Starting $\frac{T_{st}}{T_N}$	Torque: Break down $\frac{T_b}{T_N}$	Capacitor: Start μF	Capacitor: Start Vac	Capacitor: Run μF	Capacitor: Run Vac
2 Pole															
0.25	A71	2880	65.9	0.80	1.96	–	5.1	–	0.78	2.6	2.8	40	250	4	440
0.37	A71	2870	72.4	0.81	2.66	–	4.9	–	1.27	2.3	2.3	50	250	5	440
0.55	B71	2865	73	0.86	3.74	–	5.9	–	1.86	2.7	2.4	80	250	8	440
0.75	80	2900	74	0.87	4.84	–	7	–	2.45	3.2	2.8	100	250	10	440
1.1	80	2880	75.5	0.94	6.6	–	6.5	–	3.6	2.4	2.5	125	250	15	440
1.5	90S	2910	77.5	0.89	9	4.5	7	7	4.9	2.3	2.6	125	250	15	440
2.2	90L	2895	77.9	0.92	13.1	6.6	6.7	6.7	7.3	2.1	2.5	150	250	20	440
3	100L	2875	75.6	0.92	18	9	5.9	5.9	10	2.5	2.3	250	250	30	440
4	112M	2900	74.6	0.93	25	12.5	5.5	5.5	13.1	1.6	2.2	300	250	30	440
5.5	132S	2930	79.2	0.96	30	15	8	8	18	2.8	2.6	560	330	60	440
7.5	132S	2930	81.2	0.99	40	20	6.8	6.8	24.5	2.4	2.3	720	330	100	440

Fig. 2. Datasheet AC single-phase motors type CSR 90S

After reviewing the motor datasheet in Fig. 2., some parameters used for the transfer function were obtained.

Table 2. Parameter of AC single-phase Motors

Variable	Unit	Value
Nominal Voltage	V	240
Speed	Rpm	2910
Power factor	-	0,89
Current	A	9
Full load torque	Nm	4,9
Inductance	H	0,03
Coefficient of friction	Nms/rad	0,0161
Momen of Inertia	$Kg.m^2$	0,0185

B. Data Collection

From the data of several parameters obtained from the previous subsections, Table 1. presents the main parameters of the Brushless DC-Servomotor 1226 012 B type DC motor used in the Laplace transform-based mathematical modeling process. The motor has a nominal voltage of 12 V with an internal resistance of 5.45 Ω and a current of 0.054 A. The inductance value of the motor winding is 85 mH, which represents the motor's ability to withstand changes in current. The rotor inertia of 1.5×10^{-8} kg-m² indicates the level of resistance to changes in the motor's rotational speed. The nominal torque of the motor was recorded at 0.0197 Nm, while the torque constant of 4.12 mNm/A shows a linear relationship between the input current and the torque produced. In addition, the back electromotive force constant (back emf) of 0.431 mV-s/rad describes the voltage induced as a result of rotor rotation. The nominal speed of the motor is 2318 rpm, and the coefficient of viscous friction is 9.17×10^{-7} Nms/rad, which takes into account the effect of frictional force on rotor rotation. These parameters are used as the basis for constructing a first-order mathematical model and transforming it to the Laplace domain for analysis and simulation of the system response in this study.

Table 2. shows the main parameters of the CSR 90S type single-phase AC motor used in the Laplace transform-based mathematical modeling study. The motor operates at a nominal voltage of 240 V and has a rotation speed of 2910 rpm. The power factor of the motor is recorded at 0.89, which indicates the efficient use of electrical power in generating mechanical work. The current consumed by the motor under full load conditions is 9 A. In addition, the full load torque generated by this motor is 4.9 Nm. These parameters form the basis for the mathematical modeling of the single-phase AC motor dynamic system and are essential for analyzing the motor performance in the time and frequency domains through Laplace transform.

C. Data Processing

After obtaining some of the parameters required in the previous sub-chapter, then model the two motor parameters mathematically in the time domain by going through two parts, namely the electrical and mechanical parts. The mathematical model of the two parts, is obtained from the equivalent circuit of a DC motor and a single-phase AC motor. After becoming the time domain, it is then converted into the Laplace domain. The Mathematical modelling and transfer function modelling of BLDC motor and single-phase AC motor is demonstrated in this paper [3].

1. Mathematical model of Brushless DC-Servomotors type 1226 012 B

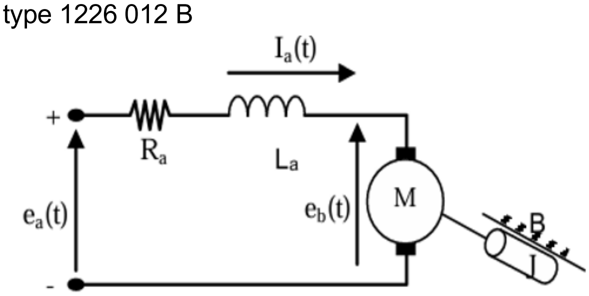

Fig. 3. DC motor equivalent circuit [13]

Explanation :

$e_a(t)$ = Input voltage
R_a = Resistance
$I_a(t)$ = Current
L_a = Inductance
$e_b(t)$ = Back emf
J = Momen of inertia

From the physical model in Fig. 3. above, it can be implemented into a mathematical model in the time domain.

Electrics section :

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$$e_a(t) = R_a.I_a(t) + L_a\frac{di_a(t)}{dt} + e_b(t) \quad (1)$$
or
$$V(t) = R.I(t) + L_a\frac{di(t)}{dt} + e(t) \quad (2)$$
Mechanic section :
$$J\left(\frac{d\omega(t)}{dt}\right) + B\omega(t) = T(t) \quad (3)$$
Motor properties :
$$T(t) = K_m I_a(t) \quad (4)$$
Generator properties :
$$e(t) = K_b\omega(t) \quad (5)$$
By using the Laplace Transformation, the time function equation above can be transformed into a Laplace equation [13]:
Electrics section :
$$e_a(s) = R_a.I_a(s) + L_a Is_a(s) + e_b(s) \quad (6)$$
Mechanic section :
$$Js\omega(s) + B\omega(s) = T(s) \quad (7)$$
Motor properties :
$$T(s) = K_m I_a(s) \quad (8)$$
Generator properties :
$$e(s) = K_b\omega(s) \quad (9)$$
After obtaining the mathematical equations of the DC motor from the time domain and laplace domain, the first-order and second-order transfer functions are obtained based on the parameters obtained previously. The first-order transfer function of a DC motor for angular speed versus input voltage is :
$$\frac{\Omega(s)}{V(s)} = \frac{K}{ts+1} \quad (10)$$
Meanwhile, the second-order transfer function of the DC motor for angular speed versus input voltage is:
$$\frac{\Omega(s)}{V(s)} = \frac{K_m}{(J_s+B)(L_s+R)+K_mK_b} \quad (11)$$
Equation (10) and Equation (11) represent the transfer functions derived from the mathematical modeling of a Brushless DC motor using Laplace transformation techniques. Equation (10) shows a first-order transfer function of the motor, where the output angular speed $\Omega(s)$ is related to the input voltage $V(s)$ by a gain constant K and a time constant t. This simplified model is useful for analyzing the basic dynamic response of the motor in control systems where high-order dynamics are negligible or can be approximated.

On the other hand, Equation (11) presents a more detailed second-order transfer function of the same system, incorporating the motor's physical parameters such as moment of inertia J, viscous friction B, armature inductance L_s, armature resistance R, torque constant K_m, and back emf constant K_b. This second-order model provides a more accurate representation of the motor's behavior by considering the combined electrical and mechanical dynamics, making it suitable for high-performance control and simulation applications. Both forms are essential in system identification, control design, and performance evaluation of electric motors in engineering analysis.

2. Mathematical model of AC single-phase motors type CSR 90S

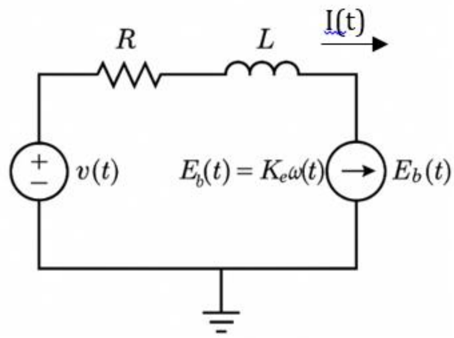

Fig. 4. Stator equivalent circuit of AC single-phase Motor

Explanation :

$V(t)$	= Input voltage
$I(t)$	= Current
R	= Coil resistance
L	= Coil inductance
$e_b(t)$	= Back emf

From the equivalent circuit of the AC motor stator, so as to obtain its electrical and mechanical equations in the time domain.
Electrics section :
$$V(t) = R.I(t) + L_a\frac{di(t)}{dt} + e(t) \quad (12)$$
Mechanic section :
$$J\left(\frac{d\omega(t)}{dt}\right) + B\omega(t) = T(t) \quad (13)$$
By using the Laplace Transformation, the time function equation above can be transformed into a Laplace equation:
Electrics section :
$$V(s) = R.I(s) + Ls.I(s) = \frac{I(s)}{V(s)} = \frac{1}{R+sL} \quad (14)$$
Mechanic section :
$$Js\omega(s) + B\omega(s) = T(s) = \frac{\omega(s)}{T(s)} = \frac{1}{Js+B} \quad (15)$$
After obtaining the mathematical equations of the single-phase AC motor from the time domain and laplace domain, the first-order and second-order transfer functions are obtained based on the parameters obtained previously. The first-order

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transfer function of a single-phase AC motor for angular speed versus input voltage is :

$$\frac{\Omega(s)}{V(s)} = \frac{K_t}{R(Js+B)} = \frac{K}{\tau s+1} \quad (16)$$

Meanwhile, the second-order transfer function of the single-phase AC motor for angular speed versus input voltage is :

$$\frac{\Omega(s)}{V(s)} = \frac{K_t}{JLs^2+(JR+BL)s+BR} \quad (17)$$

Equation (16) and Equation (17) present the first-order and second-order transfer functions, respectively, for a single-phase AC motor, specifically describing the relationship between angular speed $\Omega(s)$ and input voltage $V(s)$ in the Laplace domain. Equation (16) simplifies the motor's dynamics into a first-order system, where the gain is determined by the torque constant K_t divided by the product of resistance R and the sum of inertia J and viscous friction B. This can be rewritten in the standard first-order form $\frac{K}{\tau s+1}$ which is beneficial for basic control analysis and simplified simulation scenarios.

Equation (17), in contrast, expresses a second-order transfer function that incorporates more detailed motor characteristics. It includes the moment of inertia J, viscous friction B, and resistance R, with a quadratic polynomial in the denominator indicating second-order dynamics. The numerator still contains the torque constant K_t, which reflects the motor's ability to convert electrical energy into mechanical motion. This model captures both transient and steady-state behavior more accurately than the first-order model, making it more suitable for high-precision control and performance analysis of single-phase AC motors in dynamic environments.

III. Result

A. Accuracy

After obtaining the first-order and second-order transfer functions of the DC motor and single-phase AC motor, then the values of several parameters obtained from the datasheet of the two motors are implemented into the transfer function formula that has been obtained previously.

1. Transfer function of DC Motors

The first is to find the first-order transfer function, the parameter values that have been obtained previously are entered into the transfer function formula in Equation (10).

$$\frac{\Omega(s)}{V(s)} = \frac{K}{ts+1}$$

Before that, we will first look for the system amplifier constant (K) and its time constant (t) which are as follows:

Calculate the gain of the system :

$$K = \frac{e_b}{{K_m}^2+R_aB} \quad (18)$$

$$= \frac{0{,}000431}{(0{,}00412)^2+5{,}45\,.(9{,}17065 \text{ x } 10^{-7})}$$

$$= \frac{0{,}000431}{2{,}19724 \text{ x } 10^{-5}}$$

$$= 19{,}61$$

$$t = \frac{JR_a}{{K_m}^2+R_aB} \quad (19)$$

$$= \frac{(1{,}5 \text{ x } 10^{-8})(5{,}45)}{(0{,}00412)^2+5{,}45\,.(9{,}17065 \text{ x } 10^{-7})}$$

$$= \frac{8{,}175 \text{ x } 10^{-8}}{2{,}19274 \text{ x } 10^{-5}}$$

$$= 0{,}0037s$$

Furthermore, the results of the two parameters are entered in Equation (10) so as to obtain the first-order transfer function:

$$\frac{\Omega(s)}{V(s)} = \frac{19{,}61}{0{,}0037s+1} \quad (20)$$

Next, find the second-order transfer function, by entering the previously obtained parameter values into the transfer function formula in Equation (11).

$$\frac{\Omega(s)}{V(s)} = \frac{K_m}{(J_s+B)(L_s+R)+K_mK_b}$$

Before inputting all parameter values into the formula, the denominator is first calculated as follows :

$$(J_s+B)(L_s+R)+K_mK_b \quad (21)$$

$= (1{,}5 \text{ x } 10^{-8}s + 0{,}0000000917065)(0{,}00085s + 5{,}45) + (0{,}00412 \text{ x } 0{,}000431)$

$= (1{,}5 \text{ x } 10^{-8}.\,0{,}00085)s^2 + (1{,}5 \text{ x } 10^{-8}.\,5{,}45 + 0{,}000000917065\,.\,0{,}00085)s + (0{,}000000917065\,.\,5{,}45) + (0{,}00085\,.\,5{,}45) + 0{,}0000017757$

$= 1{,}275 \text{ x } 10^{-11}s^2 + 8{,}25295 \text{ x } 10^{-8}s + 0{,}0046391$

Furthermore, the results of the two parameters are entered in Equation (11) so as to obtain the second-order transfer function:

$$\frac{\Omega(s)}{V(s)} = \frac{0{,}00412}{1{,}275 \text{ x } 10^{-11}s^2+8{,}25295 \text{ x } 10^{-8}s+0{,}0046391} \quad (22)$$

2. Transfer function of AC Motors

In obtaining the first-order transfer function of a single-phase AC motor, the parameter values that have been obtained from the datasheet are entered into the transfer function formula in Equation (16).

$$\frac{\Omega(s)}{V(s)} = \frac{K_t}{R(Js+B)} = \frac{K}{\tau s+1}$$

But before that, we will look for the value of the system gain, which shows how much the output responds to the input signal in steady-state, and the time constant, which describes how fast the system responds to changes in input.

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$$K_t = \frac{Full\ load\ torque}{V} = \frac{4,9}{240} = 0,02 \quad (23)$$

until we get K with the formula :

$$K = \frac{K_t}{B} = \frac{0,02}{0,0161} = 1,24 \quad (24)$$

$$\tau = \frac{J}{B} = \frac{0,0185}{0,0161} = 1,149068 \quad (25)$$

After obtaining the values of the variables K and τ the two values are then entered into the first-order transfer function formula in Equation (16).

$$\frac{\Omega(s)}{V(s)} = \frac{1,24}{1,149068s+1} \quad (26)$$

Next, find the second-order transfer function, by entering the previously obtained parameter values into the transfer function formula in Equation (17).

$$\frac{\Omega(s)}{V(s)} = \frac{K_t}{JLs^2 + (JR + BL)s + BR}$$

Before inputting all parameter values into the formula, the denominator is first calculated as follows :

$$JLs^2 + (JR + BL)s + BR \quad (27)$$

$= 0,0185 \,.\, 0,03s^2 + (0,0185 \,.\, 26,6 + 0,0161 \,.\, 0,03)s + 0,0161 \,.\, 26,6$
$= 0,000555s^2 + 0,4925s + 0,428$

Furthermore, the results of the two parameters are entered in Equation (17) so as to obtain the second-order transfer function:

$$\frac{\Omega(s)}{V(s)} = \frac{0,02}{0,000555s^2+0,4925s+0,428} \quad (28)$$

B. Performance

In this study, performance evaluation is conducted based on the transfer function representations derived using the Laplace transform. Four transfer function models are considered two for the Brushless DC-Servomotor (BLDC) and two for the single-phase AC motor. Each of these models both first order and second order provides distinct perspectives on the dynamic behavior of the motors. As in getting the step response of the two motors using MATLAB/Simulink .

The result of the first-order transfer function in Equation (20) states the relationship between the motor angular velocity $\Omega(s)$ and the input voltage $V(s)$. The value 19,61 is the system gain, expressing how much the angular velocity responds to the input voltage at steady state and 0,0037 is the time constant τ, in seconds, indicating how quickly the system responds to changes in input. Sistem ini dikatakan stabil karena memiliki satu kutub real negatif di $s = -\frac{1}{0,0037} \approx -270,27$ (29). The time constant τ =0,0037 seconds indicates that the system has a very fast response. As a general rule, first-order systems reach about 95% of the final value in about 3τ, in just 0,0111 seconds. And the gain value of 19,61 means that if we give a constant input of 1 Volt, the system output (angular velocity) will stabilize at 19,61 rad/s.

Next, the result of the second-order transfer function in Equation (22) states the relationship between the motor angular velocity $\Omega(s)$ and the input voltage $V(s)$. Numerator value 0,00412 Indicates the system gain (torque constant or gain from input to output). As for the denominator, the value of $1,275 \text{ x } 10^{-11}s^2$ related to mass/inertia and inductance, the value of $8,25295 \text{ x } 10^{-8}s$ related to damping/friction and resistance, and value 0,0046391 related to the fixed effect (stiffness or counter-motion force constant). The system has two poles (roots of the denominator) that can produce different types of response underdamped (oscillating), critically damped (fast without oscillation), or overdamped (slow). Since all the coefficients are positive and small, the system is likely to be stable, but has a very fast response time as the scale values are very small. The small gain (0,00412) indicates that the steady-state response to the output will be small relative to the input.

The result of the first-order transfer function in Equation (26) states the relationship between the motor angular velocity $\Omega(s)$ and the input voltage $V(s)$. The value of 1,24 this is the gain of the K system, which indicates the magnitude of the steady-state speed response to the voltage input dan value of 1,149068 this is the τ time constant of the system, which states how fast the system reaches steady state. This system is a linear, stable, and systemized system because the poles in $s = -\frac{1}{1,149068} \approx -0,87$ (30). The time constant τ=1.149068 seconds means that the system reaches about 63% of the final value within that time and to reach about 95% of the final value, it takes about $3\tau \approx 3.45$ seconds. A gain of 1,24 means if a constant voltage of 1 voltage is input, the angular velocity output will stabilize at 1.24 rad/s.

And the last, the result of the second-order transfer function in Equation (28) states the relationship between the motor angular velocity $\Omega(s)$ and the input voltage $V(s)$. The numerator 0,02 indicates the system gain is the degree of sensitivity of the output to the input. There are 3 denominators in this transfer function : 0,000555s^2 part ineria and inductance, representing acceleration (second-order dynamics), 0,4925s damping parts (viscous damping and electrical resistance) and 0,428

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part of the system stiffness or fixed effects, such as static friction or steady-state energy conversion. The system has two poles, and its behavior can be underdamped, critically damped, or overdamped depending on the value of the damping ratio (ζ). Based on the shape of the coefficient :

$$\frac{\Omega(s)}{V(s)} = \frac{K}{as^2+bs+c} \quad (31)$$

With :

a = 0,000555

b = 0,4925

c = 0,428

Then it can be calculated natural frequency (ω_n) and damping ratio (ζ) as follows :

$$\omega_n = \sqrt{\frac{c}{a}} = \sqrt{\frac{0{,}428}{0{,}000555}} \approx 27{,}76 \quad (32)$$

$$\zeta = \frac{b}{2\sqrt{ac}} = \frac{0{,}4925}{2\sqrt{0{,}000555\ .0{,}428}} \approx \frac{0{,}4925}{2\ .0{,}0154} \approx 15{,}97 \quad (33)$$

Since ζ > 1, the system is overdamped meaning it responds slowly, without oscillation, and takes longer to reach its steady state.

Each of the four transfer function models provides a valuable approximation of the motor system dynamics. The first-order models are effective for initial analysis and fast control design, whereas the second-order models deliver more accurate insights for advanced applications [14]. The use of the Laplace transform is instrumental in transitioning from complex physical systems to functional models that can be analyzed, simulated, and optimized ultimately improving the performance and reliability of motor driven systems.

IV. Discussion

The mathematical models and corresponding transfer functions obtained from the Laplace domain have provided a solid framework for analyzing the dynamic behavior of the BLDC motor type 1226 012 B and single-phase AC motor type CSR 90S. The results from both first-order and second-order transfer function models show significant differences in terms of response characteristics such as rise time, settling time, system gain, and damping behavior.

The first-order model for the BLDC motor shows a rapid system response with a time constant of only 0,0037 seconds and a system gain of 19,61, suggesting that the motor reacts almost instantaneously to voltage changes. In contrast, the second-order model of the same motor, with a small gain of 0,00412 and significantly small coefficients in its denominator, offers a more precise but slower and lower-amplitude response. This highlights the trade-off between simplicity (first-order) and accuracy (second-order) in modeling motor systems.

Similarly, for the AC motor, the first-order model is relatively slower, with a time constant of 1,149068 seconds and a gain of 1,24. This indicates that the AC motor responds more gradually compared to the BLDC motor, aligning with its typical behavior in low-cost and less dynamic applications. The second-order model of the AC motor further illustrates the overdamped nature of the system, with a damping ratio (ζ) greater than 1 and a natural frequency (ω_n) of approximately 27,76 rad/s, resulting in a stable yet slower response curve.

The Laplace domain modeling approach bridges the gap between theoretical equations and practical control system implementation . By identifying the correct level of model complexity, engineers can optimize both computational efficiency and control accuracy.

A. Classifier

In this study, the implementation of the classifier is not aimed at categorizing image data, but rather to classify the accuracy level of the system response based on the type of transfer function. Here, a logic-based performance classifier is used that evaluates model response characteristics such as rise time, steady state time, overshoot, and steady state error to categorize motor models into performance classes such as High Accuracy, Medium Accuracy, or Low Accuracy. Each transfer function model was simulated using MATLAB/Simulink to generate the step responses [15] shown in the following table.

Table 3. Classification of performance metrics based on transfer function results

Class	Rise time (tr)	Settling time (ts)	Steady-State error (ess)
High Accuracy	tr < 0.5 s	ts < 1,5 s	ess < 5%
Moderate Accuracy	0.5 ≤ tr < 1.5	1,5 ≤ ts < 3	5% ≤ ess ≤ 10%
Low Accuracy	tr ≥ 1.5	ts ≥ 3	ess > 10%

Table 3. presents the performance classification criteria used to evaluate the quality of system responses derived from transfer function models. A system is classified as High Accuracy if it has a rise time of less than 0,5 seconds, a settling time under 1,5 seconds, and a steady-state error below 5%. These criteria indicate a fast, stable, and precise system response, ideal for applications requiring high-performance control. Systems with rise times between 0,5 and 1,5 seconds, settling times between 1,5 and 3 seconds, and steady-state errors ranging from 5% to 10% fall into the Moderate Accuracy category. These models represent a balance between responsiveness and system complexity and are suitable for general-purpose applications. Lastly, systems with rise times greater than or equal to 1,5 seconds, settling times over

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3 seconds, and steady-state errors exceeding 10% are considered Low Accuracy, indicating slow or poorly damped responses and limited suitability for precision control tasks.

B. Confusion matrices

To further evaluate the reliability of classification, a confusion matrix is constructed by comparing the expected performance class based on physical assumptions and datasheet behavior with the predicted class based on simulation results. From table 1. can be concluded the BLDC first-order model shows the best performance: very fast response τ = 0.0037s and high gain, suitable for precision control. It is therefore categorized as High Accuracy. The BLDC second order model is more complex and detailed, but has a small gain and slower response relatively speaking. It is still quite accurate, so it is classified as Moderate Accuracy. The AC motor first order model has a moderate response with a time constant greater than 1 second. This makes it fall into the Moderate Accuracy category.

And the last, the AC motor second order model has a high damping ratio ($\zeta > 1$) which causes the system to be overdamped, so it falls into the Low Accuracy category due to its longer stabilization time and low output sensitivity.

V. Conclusion

This study has applied the Laplace Transform to develop mathematical models and transfer functions for both the Brushless DC-Servomotor type 1226 012 B and the single-phase AC motor type CSR 90S. The approach included collecting physical parameters, formulating differential equations from mechanical and electrical models, and transforming them into the frequency domain to derive first-order and second-order transfer functions [16].

From the result, it can be concluded that is the first-order models provide fast, simplified analysis with acceptable accuracy for initial controller design, especially in systems where high-frequency dynamics are negligible. Second order models offer more accurate representations of motor behavior by including inductance, inertia, and damping, which are crucial for high-performance and real-time motor control applications.The BLDC motor exhibits a significantly faster and higher-gain response compared to the AC motor, making it suitable for precision motion control tasks. And the AC motor demonstrates more stable but slower dynamics, in line with its general usage in household and small industrial systems.

The use of Laplace Transform enhances the modeling process by simplifying complex time-domain differential equations into algebraic forms suitable for simulation, analysis, and control system design. The models developed can serve as a robust foundation for simulation, controller development such as PID control and future optimization in electromechanical systems.

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