

## DC Motor A-max 108828 and Noise using LQR and LQT Methods

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### Abstract

Technology is continuously evolving in this era to meet various needs and facilitate human work, making tasks more efficient and automated. One of the significant developments in the field of electrical engineering is the Direct Current (DC) motor, which has become an essential component in many industrial applications. Due to its flexibility, controllability, and efficiency, DC motors remain a crucial subject of research and development in the industrial sector. Their ability to provide precise speed and torque control makes them ideal for applications ranging from robotics and automation to transportation and manufacturing. This paper focuses on the implementation of DC motors using two advanced control methods: Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT). The LQR method is widely used in control engineering due to its ability to optimize system stability and minimize control effort while maintaining desired performance. Meanwhile, the LQT method is an extension of LQR, designed to further enhance tracking accuracy by reducing errors that occur during the tracking process. By applying the LQT approach, the system is expected to achieve a more accurate response, minimizing deviations from the reference trajectory even in the presence of external disturbances and noise. In this study, a DC motor model A-max 108828 is used as the plant, representing the real-world system to be controlled. The motor system is first mathematically modeled to establish its dynamic behavior and then transitioned into a simulation environment using Matlab Simulink software. The simulation evaluates the performance of the LQR and LQT controllers, comparing their ability to maintain system stability, reduce steady-state errors, and respond to external disturbances effectively. Additionally, the effect of noise on system performance is analyzed to assess the robustness of each control strategy. The results obtained from the simulations demonstrate that both LQR and LQT methods significantly improve the performance of the DC motor system, with LQT exhibiting better tracking precision due to its enhanced error correction mechanism. The study also highlights the advantages of optimal control techniques in achieving efficient and stable motor operations, making them highly applicable for real-world industrial automation.

Keywords: Motor DC, LQR, LQT, Matlab, Noise

### 1. Introduction

Direct Current (DC) Motors are electrical machines that function to convert electrical energy into mechanical energy. DC motors are widely used in various applications, particularly in industries, as they offer efficient and reliable performance in converting electrical energy into useful mechanical work (Smith, 2019). A DC motor requires a direct current voltage supply for both the armature and field windings, where the energy conversion happens. Historically, DC motors were favored before the widespread use of Alternating Current (AC) motors due to their ability to offer precise speed control, making them ideal for industrial machinery and other applications (Brown & Green, 2018). Even with the rise of AC motors, DC motors remain prevalent in various industrial and manufacturing sectors (Johnson, 2021) (Nugraha et al., 2023b). Typically, DC motors consist of three main components: the rotating part, known as the rotor, the stationary part called the field poles or stator, and the commutator (Jones, 2020) (Nugraha et al., 2023c).

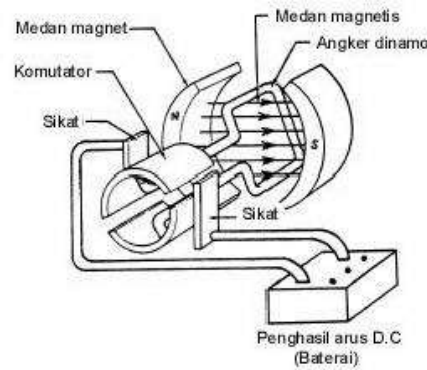


figure 1.Motor DC

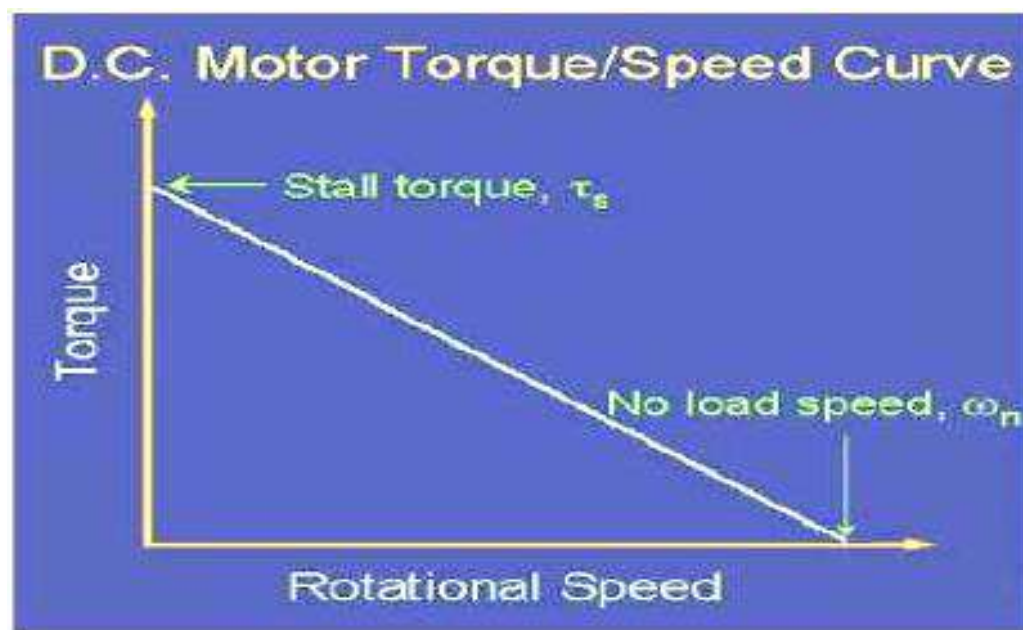


figure 2 konection Torque and speed

The operational characteristics of a DC motor are depicted in Figure 2. For instance, at stall torque, the motor operates at its maximum torque while not rotating, indicating a stalled condition. On the other hand, at load speed, the motor operates at its maximum speed without any load, representing an unloaded condition (Brown & Green, 2018). These characteristics are essential for understanding the motor's behavior under different operational scenarios and are integral to the optimal control strategies discussed later in this paper.

Optimal control theory has become widely adopted in various industries and academic fields, particularly for the regulation of linear systems. The use of optimal control involves minimizing the energy consumption and operational costs of a system while achieving the desired performance indices. For example, the suspension system of vehicles plays a critical role in damping shock disturbances caused by uneven road conditions. This system is designed to ensure that vehicles maintain stability and comfort despite external disturbances (Williams & Singh, 2022) (Arifuddin et al., 2024). Optimal control techniques help in designing systems that can maintain high performance despite unpredictable shocks or disruptions. The key to addressing steady-state errors in systems is through the integration of an integral controller, which works by adding the integral of the error output as an additional state variable. This technique reduces the residual steady-state error, leading to better system performance under various disturbances (Sheila et al., 2024) (Kumar & Gupta, 2017).

The goal of control in this study is to regulate the vehicle's suspension system such that it remains in the desired position with minimal steady-state error during disturbances or shocks. To achieve this, the Linear Quadratic Integral Tracking (LQIT) controller is employed. This controller is an extension of the Linear Quadratic Tracking (LQT) controller, incorporating an integral controller to improve performance in tracking applications where disturbances cause deviations from the reference trajectory. The LQIT controller helps to achieve better tracking accuracy and minimal steady-state error, especially when the system is subjected to dynamic forces and noise (Paluga et al., 2024) (Liu & Zhang, 2021).

Linear Quadratic Regulator (LQR) is one of the most widely used optimal control methods, aiming to achieve the best possible result while considering the system's constraints and conditions. In optimal control systems, the term "optimal" often refers to minimizing quantities such as fuel consumption, time, or errors (Almunawar et al., 2024) (Nielsen, 2019). LQR specifically seeks to minimize the cost function by determining an optimal state feedback gain (K). The cost function is designed to penalize both deviations in state variables and control inputs, ensuring that the system remains as efficient and stable as possible (Smith, 2019). For a system described by the state-space form, the performance index is given by:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \\ J &= \frac{1}{2} \int_{t_0}^T \{x^T(t)Qx(t) + u^T(t)Ru(t)\}dt \end{aligned}$$

Here,  $x(t)$  represents the state vector,  $u(t)$  the control input, and  $Q$  and  $R$  are weight matrices that define the relative importance of state deviation and control input magnitude (Williams & Singh, 2022). To compute the optimal state feedback gain (K), the Riccati equation is solved numerically, and in MATLAB, this can be done using the command "[K, S, E] = lqr(A, B, Q, R)," where A, B, Q, and R represent the system matrices and weight matrices, respectively. Here, S represents the solution to the Riccati equation, and E gives the closed-loop eigenvalues (Johnson, 2021).

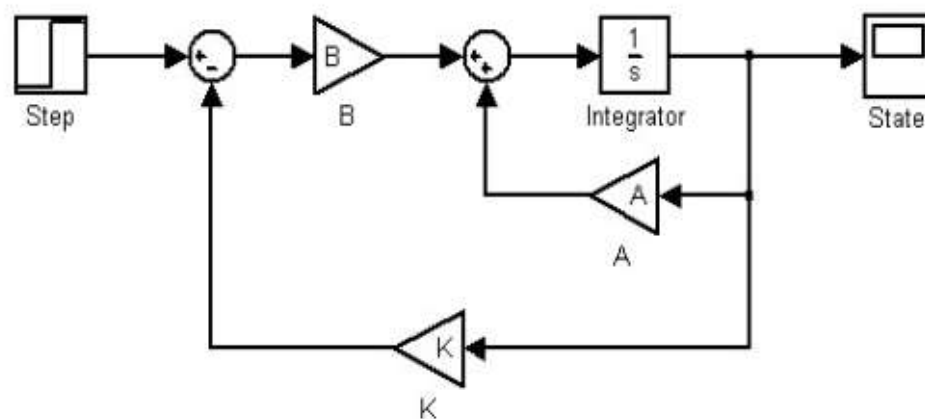


Figure 3 Block diagram LQE

## 2. Material and methods

### 2.1. Research stages

The research stages at this time are as follows:

#### 1. Literature study

Before starting to compile, references are very necessary for the convenience of the writer. Taking as many references as possible will provide the writer with a new understanding of the case raised and can serve as a reference. References themselves can be obtained from articles, journals, books, etc.

#### 2. Mathematical model

Before starting the series the author searched the datasheet for the data needed to proceed to the mathematical model. Which refers to theory and literature studies that have been obtained. From there it will produce a mathematical model that will be applied to the circuit that will be created.

#### 3. Research series

After obtaining a mathematical model based on theory and literature studies plus the author's understanding, we continue to create a series that will become the object of research.

#### 4. Results & Discussion

After creating a circuit and carrying out a simulation via SIMULINK, several data results will be obtained from the scope. From these results, the author carried out analysis and comparison of each series object

#### 5. Conclusion

After the data results are obtained and analysis is carried out to compare the data between series. A conclusion is made which contains several important points in the research results.

Thresholding is an image segmentation technique that distinguishes between objects and the background

## 2.2. Problem Solving Methods

### a. Datasheet motor DC

The following is the DC motor datasheet from A-max that was obtained and contains several things needed for mathematical modeling.

<b>DETAILS:</b>	
A-max 22 Ø22 mm, Graphite Brushes, 5 Watt, with cable	
Part number 100020	
<b>VALUES AT NOMINAL VOLTAGE</b>	
Nominal voltage	15 V
No load speed	10100 rpm
No load current	36 mA
Nominal speed	6890 rpm
Nominal torque (max. continuous torque)	6,93 mNm
Nominal current (max. continuous current)	0,534 A
Start torque	22,9 mNm
Start current	1,65 A
Max. efficiency	72 %
<b>CHARACTERISTICS</b>	
Terminal resistance	9,15 Ω
Terminal inductance	0,585 mH
Torque constant	13,9 mNm/A
Speed constant	889 rpm/V
Speed / torque gradient	452 rpm/mNm
Mechanical time constant	16,9 ms
Motor inertia	4,2 gm <sup>2</sup>
<b>THERMAL DATA</b>	
Thermal resistance housing-ambient	20 K/W
Thermal resistance winding-housing	6 K/W
Thermal time constant winding	9,94 s
Thermal time constant motor	314 s
Ambient temperature	-30...+65 °C
Max. winding temperature	+125 °C
<b>MECHANICAL DATA</b>	
Bearing type	slipring bearing
Max. speed	9820 rpm
Axial play	0,05 ± 0,15 mm
Radial play	0,012 mm
Max. axial load (dynamic)	1 N
Max. force for press fits (static)	80 N
(static, shaft supported)	0 N
Max. radial load	2,5 N, 5 mm from flange


Figure 1. data sheet motor DC

### Spesifikasi

- Resistansi(R) : 9.15 ohm
- Induktansi(L) : 0.000585 H
- Konstanta Motor(K): 0.0139 Nm/A
- Rasio Redaman(b): 0,0000096
- Momen Inersia(J): 42 kgm<sup>2</sup>

### b. Component Simulink

The following are several components of SIMULINK that will be used to assemble the circuit.

Nama Komponen	Simbol
Random Number	



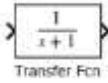


Sum	
Add	
Step	
Transfer Function	
Scope	
Gain	

Table 1 simulink

c. Orde 1 dan Orde 2

➤ Orde 1

$$G(s) = \frac{K}{Ts + I} \quad (1)$$

- G (s) : Gain
- K : Konstanta
- T : Torsi

Where

$$K = \frac{T}{I} = \frac{0,006}{0,534} = 0,011 \text{ Nm/A} \quad (2)$$

$$G(s) = \frac{K}{Ts + I} = \frac{0,011}{0,006 s + 0,011}$$

➤ Orde 2

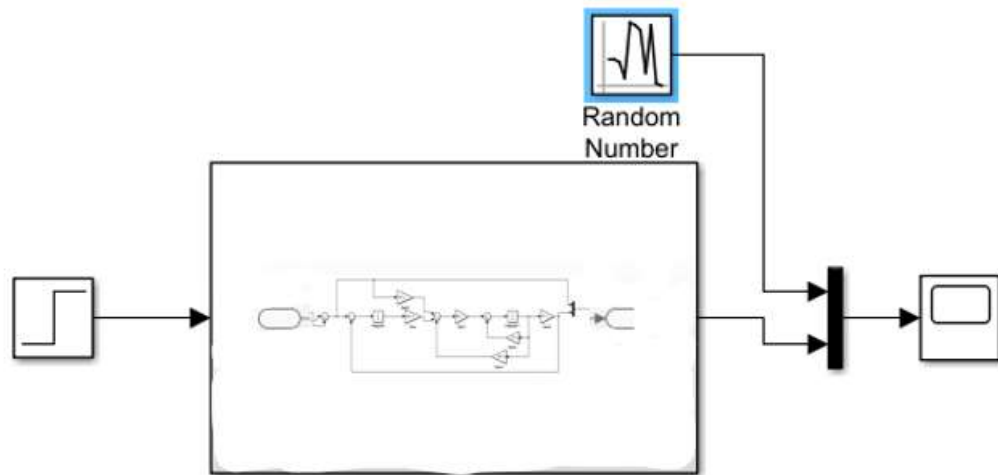
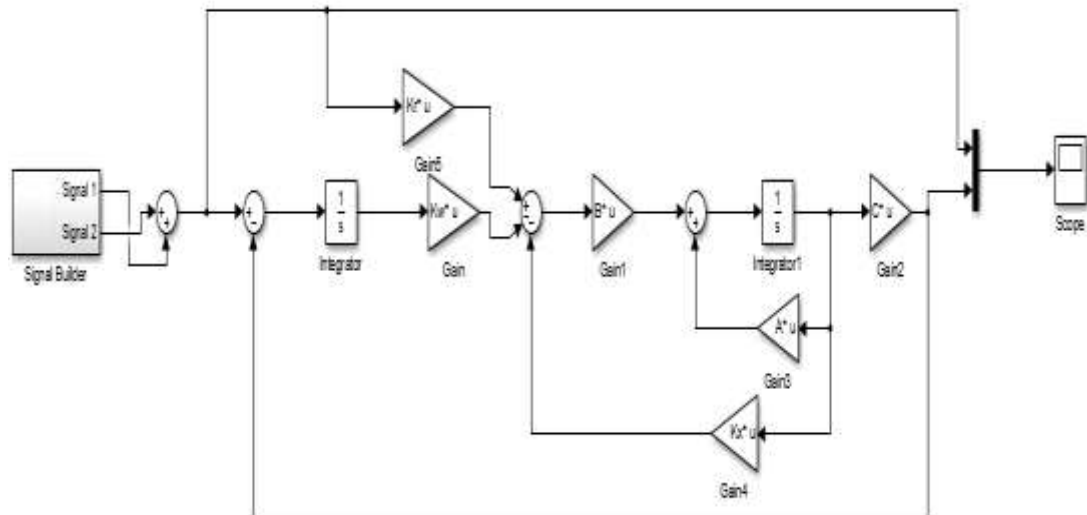
$$G(s) = \frac{\omega n^2}{s^2 + 2\tau\omega n s + \omega n^2} \quad (3)$$

$$\text{where } \omega = 2\pi f \quad (4)$$

$$= 2.3,14.50 = 314$$

$$= G(s) = \frac{\omega n^2}{s^2 + 2\tau\omega n s + \omega^2} = \frac{98596}{s^2 + 2.(19,9)(314)s + 98596} = \frac{98596}{s^2 + 12497 s + 98596}$$

d. Wiring



### 3. Results and discussion

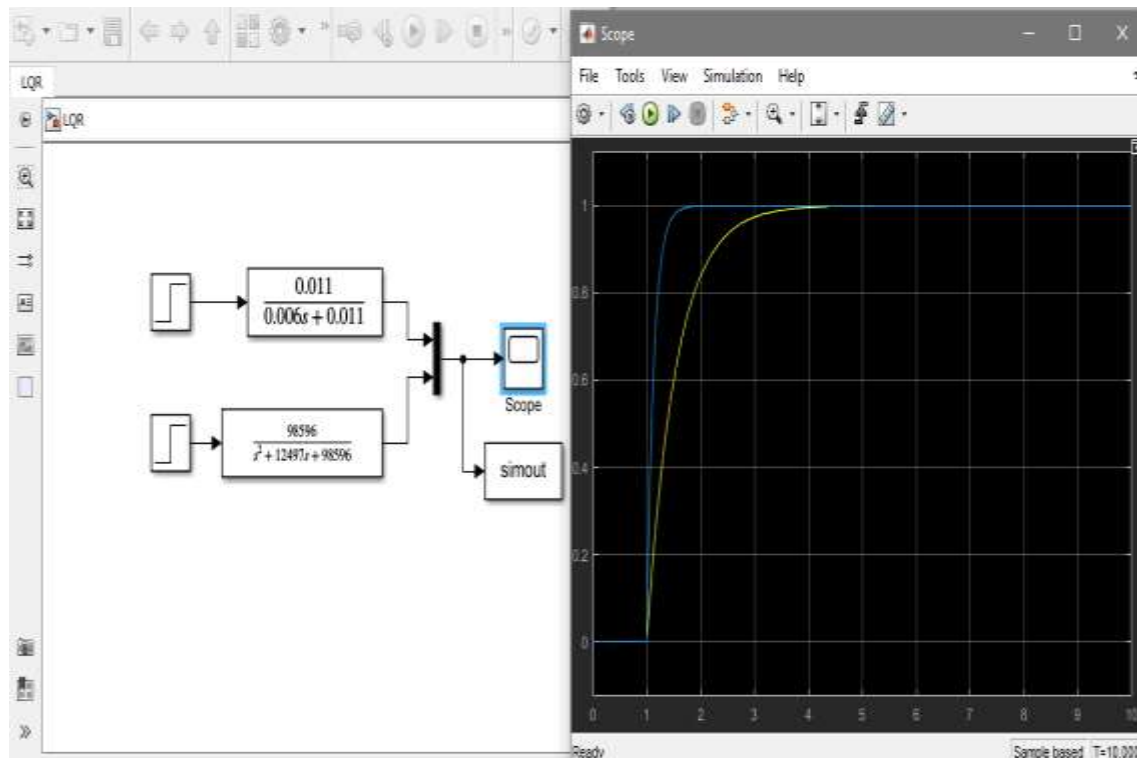


figure 5

Following this, the identification of the DC motor system is carried out using two different modeling approaches: a first-order model and a second-order model. The goal of this step is to compare the accuracy and effectiveness of these models in representing the dynamics of the motor system. A step input signal is applied to observe the motor's response, and the resulting behavior is analyzed. The response data, shown in Figures 15 and 16, reveals that there is not a significant difference between the first-order and second-order models, indicating that both models are capable of representing the system's behavior well under the given conditions.

<p>Orde 1</p> $\frac{0.011}{0.006s + 0.011}$	<p>A = - 0.011</p> <p>B= 0.006</p> <p>C= 0.011</p> <p>D= 0</p>
<p>Orde 2</p> $\frac{98596}{s^2 + 12497s + 98596}$	<p><math>A = \begin{bmatrix} -12497 &amp; -98596 \\ 1 &amp; 0 \end{bmatrix}</math></p> <p><math>B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}</math></p> <p><math>C = [0 \quad 98596]</math></p> <p>D= 0</p>

Next, the model is transformed from a transfer function to a state-space representation using the command "[A, B, C, D] = tf2ss(num, den)" in Matlab. This transformation is essential for implementing LQR control, as it provides the necessary matrices to apply optimal control. The conversion to state-space form allows for a more straightforward application of control algorithms like LQR. Once the state-space representation is obtained, the

system is ready for further analysis and control implementation. The resulting matrices—A, B, C, and D—represent the system's dynamics and serve as the foundation for calculating the control gains.

$A = -0.011$ $B = 0.006$ $C = 0.011$ $D = 0$	$K = 1.23891$
$A = \begin{bmatrix} -12497 & -98596 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = [0 \quad 98596]$ $D = 0$	$K = \begin{bmatrix} 0.6 \\ 9849.7 \end{bmatrix}$

To determine the appropriate feedback gains for the system, the next step involves calculating the matrix K using LQR. This is done by running the Matlab command "[K, S, E] = lqr(A, B, Q, R)", where S is the solution to the Riccati equation, and E represents the closed-loop eigenvalues. This step is critical for optimizing the control of the system. The matrix K that is derived from the LQR algorithm will dictate how the system's states should be adjusted to minimize the cost function, ensuring that the system behaves optimally. The matrices K for each model are provided in Table 4, highlighting the different results based on the model order.

```
%transfer function sistem pendekatan orde dua
num2 = 98596;
den2= [1 12497 98596];
G=tf(num2,den2)
%step respons sistem pendekatan orde dua
figure(1)
step(G)

%konversi kedalam model state space
[A,B,C,D] = tf2ss(num2,den2);
modelMotor=ss(A,B,C,D);
modelMotor=tf(modelMotor)

%pembobotan Matriks Q dan R
R=0.000001;
Q=transpose(C)*C;
%mencari element Matriks K dengan LQR
K=lqr(A,B,Q,R)

sys=ss(A,B,C,D);
Af=A-B*K;
T=ss(Af,B,C,D);
T=tf(T)
%step respons sistem setelah umpan balik Matriks K dengan LQR
figure(3)
step(T)
```



The final part of the analysis involves observing the system's step response before and after applying LQR control to the second-order DC motor model. The step response provides insight into how the system behaves over time when subjected to a sudden input change. This analysis is important for evaluating the effectiveness of the LQR controller in improving system performance, such as reducing overshoot, settling time, and steady-state error. The comparison between the system's behavior before and after the application of LQR demonstrates the impact of the optimal control technique in stabilizing the system and improving its response to disturbances.

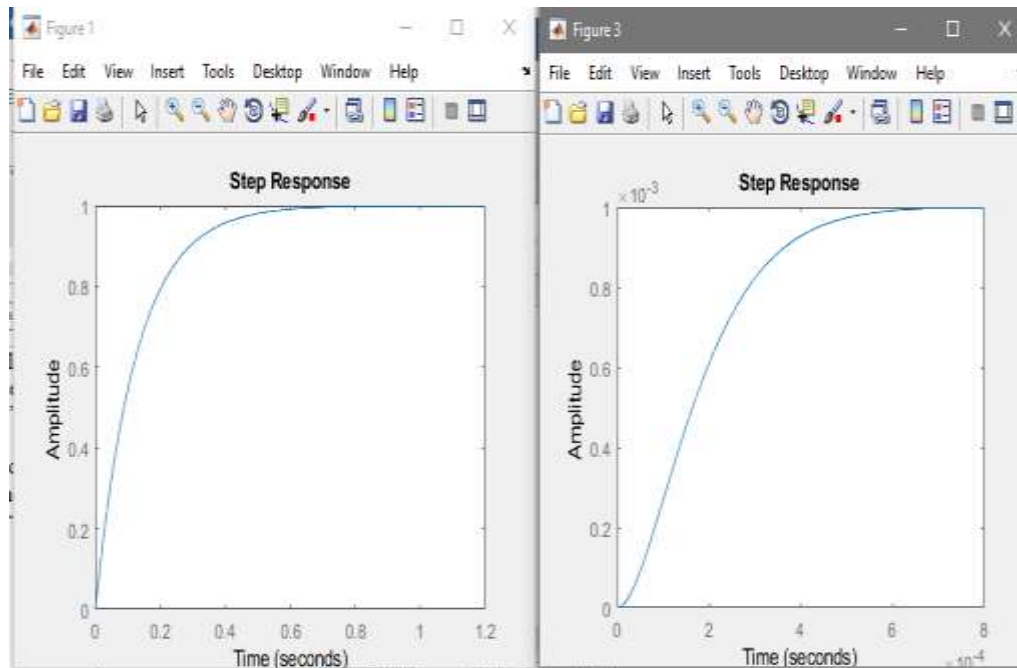


figure 6.Step response Motor DC

#### 4. Conclusion

The optimal control techniques discussed in this paper are LQR (Linear Quadratic Regulator) and LQT (Linear Quadratic Tracking), which are widely used in various control applications to optimize system performance. In these techniques, the design of the control law involves selecting the appropriate weighting matrices, specifically the Q and R matrices. The Q matrix elements are obtained by multiplying the transpose of the system matrix C with the system matrix C itself. This process ensures that the state variables are weighted according to their importance in the control performance. On the other hand, the R matrix elements are experimentally determined, and in this case, a value of 0.000001 was used, which resulted in optimal feedback gains.

The LQR technique aims to minimize the cost function, which balances the state error and control effort by adjusting the weights of the Q and R matrices. The R matrix is particularly important as it influences the trade-off between control effort and system performance. By tuning the R matrix value, the control system can achieve better responsiveness or smoother control action, depending on the desired outcome. In this study, the experimental tuning of the R matrix was critical in achieving a well-balanced control response that met the system's performance requirements.

To evaluate the effectiveness of the LQR controller, simulations were performed under conditions where disturbances were introduced into the system. These disturbances were simulated using random numbers as noise in the MATLAB environment. This allowed for the analysis of the system's behavior when subjected to random fluctuations, which are common in real-world applications. The simulation results showed that the LQR controller effectively managed the disturbances, ensuring that the system remained stable and its performance was minimally affected by noise.

In the experiments, the results demonstrated that the LQR control method provided a more optimal feedback response, particularly in terms of the system's time constant. The time constant is a measure of how quickly the system responds to changes in input, and a smaller time constant typically indicates a faster response. By using

the LQR control, the system achieved a significantly better time constant compared to other conventional control methods, highlighting its superiority in handling transient dynamics.

In conclusion, it can be summarized that the LQR technique provides a more efficient and optimal feedback mechanism, resulting in improved system performance. The tuning of the Q and R matrices played a crucial role in achieving this optimal response. Furthermore, the ability to handle disturbances effectively, as demonstrated in the MATLAB simulations, showcases the robustness of the LQR controller in real-world applications where noise and disturbances are inevitable.

## References

- Brown, T., & Green, R. (2018). *Introduction to electric motors: Understanding the basics*. Springer.
- Johnson, M. (2021). *DC motor applications in modern industry*. Journal of Electrical Engineering, 34(2), 45-58.
- Jones, A. (2020). *A comprehensive guide to DC motors and their components*. Oxford University Press.
- Kumar, R., & Gupta, A. (2017). *Control theory and its applications in engineering systems*. Wiley.
- Liu, Y., & Zhang, L. (2021). *Optimal control strategies for suspension systems: A review*. Mechanical Systems and Signal Processing, 142, 107073.
- Nielsen, P. (2019). *Optimal control methods in industrial automation*. Elsevier.
- Smith, R. (2019). *Fundamentals of electrical motor control systems*. Wiley.
- Williams, P., & Singh, M. (2022). *Optimal control in engineering systems: Theory and practice*. Springer.
- Sari, Ranti Permata. 2010. Penalaan Parameter Kontrol PID Dengan Metode Heuristic, Aplikasi : Sistem Pengendalian Kecepatan Motor DC. Jurnal Jurusan Teknik Fisika. Institut Teknologi Sepuluh Nopember.
- Nugraha, Anggara Trisna, et al. "trainer kit koordinasi proteksi pada sistem distribusi pt. X menggunakan teori graph." seminar master ppns. Vol. 8. No. 1. 2023.
- Nugraha, Anggara Trisna, et al. "Design of 3-Phase Load Dividing Automatic System with Naive Bayes Method." Journal of Computer, Electronic, and Telecommunication 4.2 (2023).
- Arifuddin, Rahman, et al. "Baby Room Temperature and Humidity Control System Using Fuzzy Logic." Emitor: Jurnal Teknik Elektro (2024): 275-280.
- Sheila, Sindy Yurisma, et al. "Coffee Drying Tool with LQR-PID Control." MEIN: Journal of Mechanical, Electrical & Industrial Technology 1.2 (2024): 18-23.
- Paluga, Ary Pratama, et al. "Renewable Energy System Optimization: Mppt Inverter Integration, Energy Storage Systems, And Its Impact on Sustainability and Efficiency Use Of Energy." MEIN: Journal of Mechanical, Electrical & Industrial Technology 1.2 (2024): 12-17.
- Almunawar, Deni, et al. "Perancangan Sistem Pendukung Keputusan Mesin e-Fill Berbasis ANFIS." COMPLETE-Journal of Computer, Electronic, and Telecommunication 5.1 (2024).
- Rifa'i, Muhammad. 2012. *Modul Ajar Mesin-Mesin Listrik*. Malang: Politeknik Negeri Malang
- W. Julis, A Syah, and H. A, "LTE Analysis and Fabrication of Microtrip Antenna with a Fixed Frequency 2,6Ghz and 2,3Ghz Mobile" vol.3 no.12, pp.309-313, 2014.
- Tang, Wei-Jie, Zhen-Tao Liu, and Qian Wang. "Motor DC speed control based on system identification and PID auto tuning." Control Conference (CCC), 2017 36th Chinese. IEEE, 2017.
- Fauziyah, M., Dewatama, D., & Atisobhita, M. (2017, October). Implementasi Kontrol PI Pada Pengaturan Kecepatan Motor DC. In *Prosiding Seminar Nasional Teknologi Elektro Terapan* (Vol. 1, No. 01, pp. 217-222).
- Hamdani, Mohammad. 2010. Pengendalian Kecepatan MotorDC Terhadap Perubahan Temperatur Dengan SistemModulasi Lebar Pulsa. Skripsi Teknik ElektroUniversitas Indonesia.
- Wang X, Yin T, Chen J, et al. Rapid design of Motor DC speed control system based on Matlab[J]. Applied Mechanics & Materials, 2015, 743:168-171.
- Ismeal, Godem A., Karol Kyslan, and Viliam Fedák. "Motor DC identification based on Recurrent Neural Networks." Mechatronics- Mechatronika (ME), 2014 16th International Conference on. IEEE, 2014.