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Comparison of LQR and LQT Control of Uncertain Nonlinear Systems

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Abstract

Optimal controls have been applied in this time. One of simple optimal control which will be analyzed in this research is planar arm model dynamic. Linear quadratic regulator (LQR) and linear quadratic tracker (LQT) are two well-known control techniques that have been widely applied in various fields such as engineering, robotics, and economics. LQR is a control method that aims to minimize the cost function of a linear system by adjusting the control input. On the other hand, LQT is a control technique that aims to track the reference input of a linear system while minimizing the cost function. Both LQR and LQT are optimal control methods that have good stability and performance characteristics. In this paper, we present a comparison of LQR and LQT control techniques and discuss their applications in various domains. We also highlight the advantages and limitations of these methods and suggest some possible future research directions.

Keywords : linear quadratic regulator, linear quadratic tracker, optimal control, stability, performance, engineering, robotics, economics.

1. Introduction

A novel LQR controller design technique was developed in this research to improve the performance of control systems (Hassani & Lee, 2014). Linear Quadratic Regulation (LQR) is a widely known method in control systems, particularly in the context of optimal control for dynamic systems. The concept of LQR revolves around minimizing a quadratic cost function that penalizes deviations from desired system states and input signals (Karthick et al., 2016). This method is applicable to both single and multi-variable systems, making it a versatile choice in various engineering fields, including robotics and electrical motor control (Herlambang et al., 2020) (Nugraha et al., 2022).

In this study, the developed LQR method was applied to control an electric motor model, specifically a quadrotor system, to evaluate its performance. The results from simulations showed that the quadrotor was capable of following changes in set-points with great accuracy and stability. This confirms that the LQR controller is effective in handling dynamic changes and disturbances within a controlled environment (Kirkpatrick et al., 1983). The ability to accurately track set-points is crucial in systems that require high precision and reliability (Putra et al., 2019).

However, one of the challenges faced in this research was the method of selecting the Q and R weight matrices, which are critical parameters in the LQR design process (Studi et al., 2021) (Rates, n.d.). The approach used in this study for determining these values was through a trial-and-error process. This method, while effective, is time-consuming and lacks a systematic approach to optimizing the matrices. Various approaches have been proposed in the literature to optimize the selection of these matrices, but a universally accepted method remains elusive (Nugraha et al., 2023).

The trial-and-error method, though it produces satisfactory results, requires significant time and computational resources to determine the optimal values of Q and R (Hidayana et al., 2024). This issue becomes particularly evident when applied to systems with many variables, where the matrix sizes grow and the computational complexity increases. Therefore, there is a need for a more efficient method to determine the optimal weight matrices for the LQR controller. Several optimization algorithms, including genetic algorithms, particle swarm optimization, and gradient-based methods, have been explored as potential solutions to this problem (Rahman et al., 2023).

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To address the aforementioned challenge, future work should focus on developing a more systematic and efficient approach for selecting the Q and R matrices in LQR design (Handandi et al., 2023). Such approaches would significantly reduce the trial-and-error effort while maintaining the performance levels achieved by the LQR controller (Kurniawan et al., 2023). Machine learning and artificial intelligence techniques could also be explored as alternative methods for optimizing these matrices based on system performance data.

In conclusion, while the LQR method applied in this research provided effective results, the manual tuning of the Q and R matrices remains a challenge. A more efficient approach for selecting these matrices would enhance the applicability of LQR in complex systems. This area of research continues to be a topic of interest, with ongoing efforts to develop smarter algorithms that can optimize these parameters in real-time

2. Material and methods

2.1. basic theory

Linear Quadratic tracking (LQT)

Linear Quadratic Tracking (LQT) is a linear control system designed to regulate the system output so that it follows a predefined trajectory, minimizing control energy (Debogoswami, n.d.). The LQT approach aims to adjust the system's output to match the desired output with minimal control effort. The underlying system is described by Equation (1), which is a linear observable system. The state-space equation for this system is given as:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)\ldots(1)$$
$$y(t) = C(t)x(t)$$

Here, x(t)x(t)x(t) represents the state vector, u(t)u(t)u(t) is the input, and y(t)y(t)y(t) is the output of the system. The error vector e(t)e(t)e(t) is defined in Equation (2), where the error is the difference between the desired output z(t)z(t)z(t) and the actual system output y(t)y(t)y(t). In order to minimize this error, a matrix P(t)P(t)P(t) is defined, which must satisfy the Riccati differential equation (Equation 3):

$$e(t) = z(t) - y(t)\ldots(2)$$

Alternatively, this can be written as:

$$\dot{P}(t) = -P(t)A(t) - A'(t)P(t) + P(t)B(t)R{-}1(t)B'(t)P(t) + C'(t)Q(t)C(t)\ldots(3)$$

atau bisa ditulis :

$$0 = -P(t)A(t) - A'(t)P(t) + P(t)B(t)R{-}1(t)B'(t)P(t) + C'(t)Q(t)C(t)\ldots(4)$$

The matrices Q(t)Q(t)Q(t) and R(t)R(t)R(t) are assumed to correspond to the desired performance of the system. Upon solving the Riccati equation, a non-homogeneous differential equation for the vector g(t)g(t)g(t) can be derived, as shown in Equation (5):

$$\dot{g}(t) = -[A - B(t)R{-}1B'(t)P(t)]'\, g(t) - C'(t)Q(t)z(t)\ldots(5)$$

Once P(t)P(t)P(t) and g(t)g(t)g(t) are determined, the final step is to compute the control gain K(t)K(t)K(t), which can be obtained from Equation (6):

K(t)=R−1(t)B′(t)P(t)(6)K(t) = R^{-1}(t)B'(t)P(t) \quad \text{(6)}K(t)=R−1(t)B′(t)P(t)(6)

With all necessary parameters calculated, the optimal control input u∗(t)u^*(t)u∗(t) can be determined using Equation (7):

$$K(t) = R{-}1(t)B'(t)P(t)\ldots(6)$$

Here, x∗(t)x^*(t)x∗(t) is the feedback from the system. The feedback is multiplied by the control gain

$$u{*}(t) = -K(t)x{*}(t) + R{-}1(t)B'(t)g(t)\ldots(7)$$

which serves as the input to the system to achieve the desired output. This approach ensures that the system's behavior is effectively controlled with minimal energy usage, thereby optimizing system performance.

Linear Quadratic Regulator (LQR) is an optimal control method widely used across various fields such as industrial applications, robotics, and engineering (Mickky & Tiwari, 2015). The advantage of LQR lies in its ability to provide an optimal solution for controlling systems defined in state-space form. The method is particularly effective in managing Multi-Input, Multi-Output (MIMO) systems, where multiple inputs and outputs need to be regulated simultaneously. To achieve the desired control action, two critical parameters—weight matrices Q and R need to be selected. These matrices influence the dynamic behavior of the closed-loop system and directly affect the performance of the control system.

Unlike Proportional-Integral-Derivative (PID) controllers, which have systematic tuning methods such as Ziegler-Nichols and Cohen-Coon, the LQR controller does not have a dedicated tuning method for selecting the weight matrices Q and R (Obeid Ahmed, n.d.). The relationship between these matrices and the dynamics of the closed-loop system is complex, making it difficult to predict the effects of different combinations of QQQ and RRR on system performance. Consequently, practitioners often resort to trial and error by testing various combinations within a certain range and selecting the one that yields the desired dynamic response (Hua et al., 2020). This process emphasizes the need for optimization techniques to fine-tune these matrices.

In this context, metaheuristic algorithms are commonly used to optimize the selection of QQQ and RRR matrices. These algorithms are inspired by natural phenomena or biological processes and are particularly suited for solving complex problems in large search spaces (Dwivedi & Dohare, 2015). One such metaheuristic algorithm is Stochastic Fractal Search (SFS), a recently developed technique that has demonstrated superior performance over other optimization methods, such as Particle Swarm Optimization (PSO), Cuckoo Search (CS), Gravitational Search Algorithm (GSA), and Artificial Bee Colony (ABC) (Shrivastva & Singh, 2014). SFS is advantageous not only due to its high accuracy and relatively short convergence time but also because it is easy to implement.

DC Motors are widely used in various applications due to their ability to convert electrical energy into mechanical motion. These motors operate using Direct Current (DC) power, requiring a constant voltage input to drive the motor's rotor (Fang, 2014). DC motors are commonly found in devices like mobile phone vibrators, DC fans, and power drills, where precise control of motion is essential. The main advantage of DC motors is their ability to provide smooth and variable speed control, making them suitable for applications where speed regulation is crucial.

Single Input, Single Output (SISO) systems are the simplest communication systems, consisting of one antenna at the transmitter and one antenna at the receiver. In such systems, the transmitted data must be modulated before being sent and demodulated upon reception to ensure accurate communication (Herlambang et al., 2020). Although simple, SISO systems are limited in terms of their ability to handle interference and noise, especially in environments with multiple signal paths, making them less robust compared to more advanced systems.

In contrast, Multiple Input, Single Output (MISO) systems use multiple antennas at the transmitter side to combat multipath fading, a common issue in wireless communication systems (Xu et al., 2019). MISO systems are a form of spatial diversity that improve signal reliability by utilizing multiple signal paths, helping to mitigate the effects of signal fading. These systems are typically used in conjunction with techniques like Orthogonal Frequency-Division Multiplexing (OFDM), which enhances data transmission efficiency and reduces Inter-Symbol Interference (ISI). MISO systems offer significant advantages in terms of signal quality and data throughput, especially in challenging communication environments.

Single Input, Multiple Output (SIMO) systems employ multiple antennas at the receiver side to enhance the reception of signals. The key challenge in SIMO systems is the design of the receiver, which must incorporate combining techniques such as Selection Combining (SC), Equal Gain Combining (EGC), and Maximal Ratio Combining (MRC) (Gawthrop et al., 2014). These methods improve signal quality by combining the received signals, with EGC and MRC using weighted summation based on the channel's phase and magnitude, respectively. SIMO systems are useful in environments where signal quality is a concern, allowing for more reliable data reception.

Finally, Multiple Input, Multiple Output (MIMO) systems leverage multiple antennas at both the transmitter and receiver to improve communication performance. MIMO technology allows for simultaneous transmission of multiple signals over the same frequency band, significantly increasing data capacity and enhancing system reliability. MIMO systems are widely used in modern wireless communication standards, such as 4G and 5G, where high data rates and reliable connections are essential. The use of multiple antennas enables MIMO systems to take advantage of spatial diversity, providing robust communication even in complex environments with high interference.

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2.2. Simulation stages

Table 1. DC motor parameters

DC Motor Parameters	Motor 1	Motor 2	Motor 3
Armature Resistance Ra (Ohm)	2	2	1
Armature Inductance La (H)	0.5	0.5	0.5
Moment of inertia J (Kgm2)	0.02	1.2	0.01
Friction constant B (Nms)	0.2	0.2	0.00003
Torque constant KT (Nm/A)	0.015	0.2	0.023
EMF constant Kb (Vs/rad)	0.01	0.2	0.023

Parametes	**Notation**	**Value**
Length of shoulder joint (m)Length of elbow joint (m)	$L_1 L_2$	0.298 0.419
Mass of shoulder joint (kg)Mass of elbow joint (kg)	m_1 m_2	2.089 1.912
Center of mass of shoulder joint (m)Center of mass of elbow joint (m)	L_{g1} L_{g2}	0.152 0.181
Moment of inertia of shoulder joint (kg m^2)Moment of inertia of elbow joint (kg m^2)	I_1 I_2	0.0159 0.0257

2.3. component

To facilitate analysis in problem-solving methods, the transfer function table can serve as a reference. This table summarizes the essential components required in a system or circuit. It provides a clear overview of how the system's input and output relationships are modeled through the transfer function. By using this table, one can identify the key parameters influencing the system's behavior and assess the system's stability and performance.

The transfer function table organizes different system configurations, outlining their specific components. This serves as a guideline to design or evaluate a circuit, ensuring all necessary elements are included. These components typically include the system's gain, poles, zeros, and any feedback mechanisms. With these components in place, it becomes easier to develop an accurate representation of the system's dynamics.

Moreover, the transfer function table helps streamline the process of solving complex systems, whether in electrical, mechanical, or other engineering fields. By using it as a reference, engineers can efficiently determine the appropriate control strategies and analyze the system's response to various inputs. This approach is particularly useful when dealing with multiple components and dynamic systems that require a structured method for evaluation and optimization.

Tabel 2. Transfer Function Motor DC

No	**Bagian Rangkaian**	**Transfer Fungsi**
1	**Orde 1 Motor**	$\frac{9,13}{4.2s + 9,13}$
2	**Orde 2 Motor**	$\frac{98596}{s^2 + 21352s + 98596}$

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a. LQT

To simulate the FTC scheme in MATLAB, a continuous-time linear DC motor model is employed . This model represents the dynamics of the motor, which is used as the plant in the feedback control system. The linear model simplifies the complexities of the motor's behavior and makes it easier to implement the necessary control strategies, particularly in tracking applications. In this approach, the DC motor serves as the physical system whose performance is analyzed through various control methods.

The proposed LQT method introduces the possibility of incorporating the time derivatives of tracking error up to order (m-1) into the cost function. This extension is crucial because it enhances the controller's ability to reduce errors over time, especially when the system's performance is sensitive to how quickly errors are corrected. By including higher-order derivatives of the error, the LQT method can provide more precise control, leading to better tracking accuracy and system stability.

In the context of the control strategy, both the LQR and LQT controllers are used to achieve the desired system behavior. The LQR method, known for its optimal performance in linear systems, minimizes the quadratic cost function by adjusting the feedback gains. Meanwhile, the LQT method builds on this by taking the time derivatives of the error into account, offering a more refined approach to tracking control, especially for systems that require high precision over time.

By integrating these advanced control techniques, the simulation aims to showcase the performance of the FTC scheme in achieving optimal tracking control. The MATLAB model allows for the testing and tuning of the system's response to different input signals and disturbance conditions. The flexibility of MATLAB's simulation environment makes it an ideal tool for analyzing and fine-tuning the performance of the LQR and LQT controllers.

The overall goal of this simulation is to compare the effectiveness of LQR and LQT in controlling the linear DC motor. By examining the tracking errors and system response, the simulation provides valuable insights into the strengths and limitations of each controller. This analysis is essential for determining which method offers the best performance under various operating conditions, thereby guiding the design of more efficient control systems in practical applications.

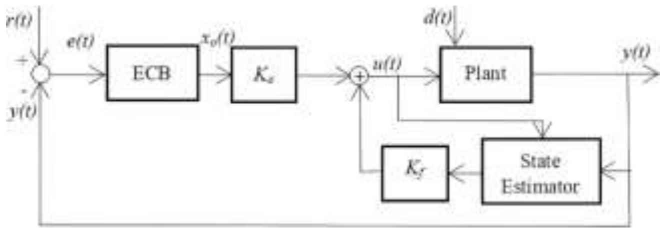

Figure 1. Diagram Block

b. LQR

The analysis of such systems is performed using the state-space approach. This approach is particularly beneficial due to its simplicity and effectiveness in representing dynamic systems. By employing state-space models, the behavior of systems can be described in terms of their states, which are related to the inputs and outputs through linear equations(Nugraha, 2023). This method offers a structured way to analyze complex systems, making it easier to design controllers that ensure desired system performance.

State-space methods are particularly useful for multi-input multi-output (MIMO) systems, where multiple inputs influence multiple outputs. This approach allows for the design of control systems that can handle the interdependencies between various inputs and outputs in a coordinated manner. The state-space representation simplifies the complexity of MIMO systems, making it easier to apply advanced control techniques, such as Linear Quadratic Regulator (LQR), for optimal performance(Nugraha et al, 2021).

Linear Quadratic Regulator (LQR) design is widely used in various applications, especially in systems described by state-space models. This technique minimizes a cost function that typically represents the trade-off between system performance and control effort (Nugraha et al, 2024b; Nugraha et al, 2024c). LQR has been extensively applied to control MIMO systems, providing optimal solutions that balance stability, response time, and control energy. Its application in control design helps achieve optimal performance in diverse engineering fields, from aerospace to robotics.

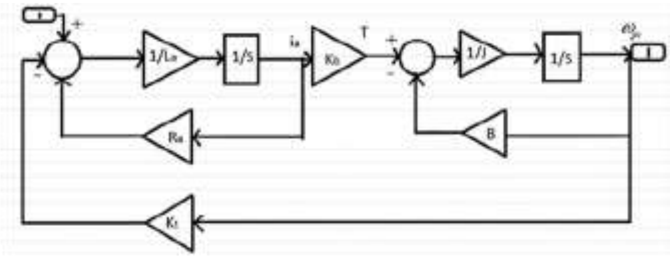

Figure 2. Simulink

```
syms x1 x2 k km m g u
x1d=x2;
x2d=g-(km*k^2*u^2/ (m*x1^2))
f=[x1d;x2d]
x=[x1,x2];
U=u
A=jacobian(f,x)
B=jacobian(f,U)
C=[ 0 1]
D=0
x1_= 0.09;
x2_=0;
m_=20.42*10^-3;
g_=9.81;
km_=2.5364*10^-5;
k_=1/0.9524;
u_=x1_*sqrt(m_*g_/(km_*k_^2));
A1=subs(A,[x1 x2 k km m g u], [x1_ x2_ k_ km_ m_ g_ u_]);
B1=subs(B,[x1 x2 k km m g u], [x1_ x2_ k_ km_ m_ g_ u_]);
A=double(A1);
B=double(B1);
sys1=ss(A,B,C,D)
[n,d]=ss2tf(A,B,C,D)
sys2=tf(n,d)
%Design LQR
N=0
Q=eye(2)*3
R=15
[Kc,sc,ec]=lqr(A,B,Q,R,N)
Ns=-1*inv(C*inv(A-B*Kc)*B)
```

3. Result and Discusiion

***Linear Quadratic Tracking* (LQT)**

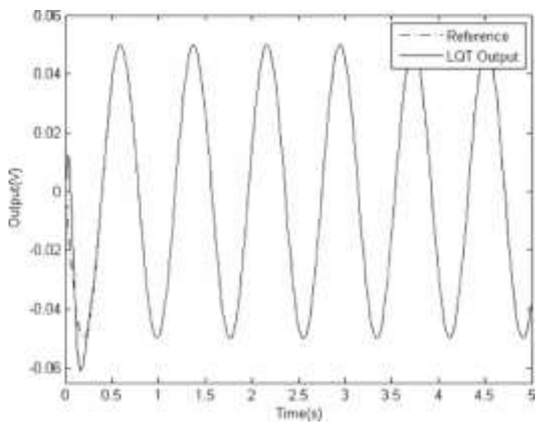

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Figure 3. LQT Waveform result

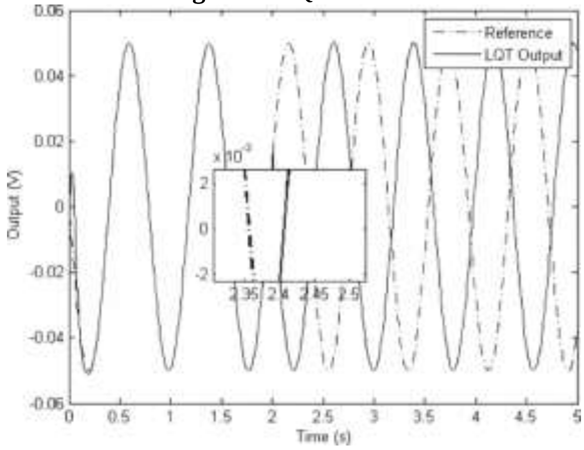

Figure 4. LQT Waveform zoom in result

***Linear Quadratic Regulator* (LQR)**

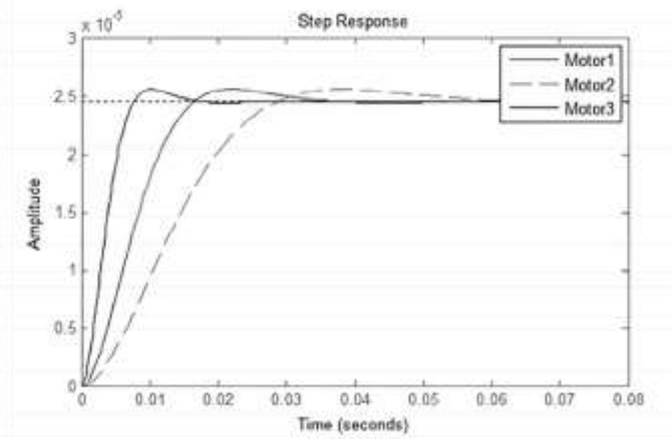

Figure 5. LQR Waveform result

Table 3. Result comparasion

Motor	*Rise time (s)*	*Max. Overshoot (percent)*	*Settling time (s)*
Motor 1	**0.016**	**4.30**	**0.0295**
Motor 2	**0.0184**	**4.31**	**0.0511**
Motor 3	**0.00496**	**4.32**	**0.0138**

3. Conclusion

The simulation results prove that the LQR controller offers better dynamic performance for DC motors in low power applications, namely motor 3, in terms of rise time and settling time. Based on the results, this can lead to a significant increase in the influence of jitter on closed-loop response and transport delays in applications such as cruise control, process control, etc. More research on the selection of Q and R in LQR methods could prove to be significant in understanding the transient response of generic systems. In LQT, the performance index value depends on the weight matrix so we have to optimize the weight matrix. Based on the simulation results, Simulated Annealing can optimize the weight matrix in the LQT so as to produce an optimal performance index with angle as the state solution following the reference and also obtain optimal control of the six muscle forces applied.

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