

Modeling of LQR and LQT Control on DC Motor C34-L-60

Abimanyu Manap¹, Abdul Hazim²

¹ Marine Electrical Engineering, Shipbuilding Institute of Polytechnic Surabaya
(abimanyumanap@student.ppns.ac.id)

² Marine Electrical Engineering, Shipbuilding Institute of Polytechnic Surabaya

Abstract

Control systems play a crucial role in the industrial world, ensuring optimal performance of a plant or process. A well-designed control system is characterized by its ability to produce a responsive and accurate output that closely aligns with the desired setpoint. This responsiveness is essential for maintaining efficiency and precision in industrial operations. In this research, the control methodology begins with the modeling of the system using MATLAB software. The study focuses on two advanced control techniques: Linear Quadratic Regulator (LQR) and Linear Quadratic Tracker (LQT). These techniques are applied to a DC Motor model, specifically the C34-L-60 motor, which serves as the primary plant in this investigation. The system modeling process involves creating a detailed representation of the motor dynamics and integrating the control algorithms to simulate its performance. By leveraging MATLAB's powerful simulation tools, the study aims to derive optimal control parameters for the motor under various operating conditions. This step is critical to achieving a comprehensive understanding of how each control method influences the system's behavior. Once the models are established, simulations are conducted to evaluate the performance of both the LQR and LQT control strategies. The simulations allow for a detailed comparison of their respective responses to the desired setpoints. Key performance metrics such as settling time, overshoot, and steady-state error are analyzed to determine the effectiveness of each control method. The data obtained from these simulations highlights the differences between the LQR and LQT methods. LQR focuses on minimizing the quadratic cost function for state variables and control inputs, while LQT extends this approach by considering tracking performance for specific reference trajectories. These distinctions provide valuable insights into the applicability of each technique for different industrial scenarios. Overall, this research emphasizes the importance of selecting an appropriate control method to achieve optimal system performance. By comparing the simulation results of LQR and LQT, the study contributes to a deeper understanding of their capabilities and limitations in controlling DC motors. This knowledge serves as a foundation for further advancements in industrial control system design.

Keywords: LQR, LQT, DC Motor

1. Introduction

Control systems play a crucial role in managing, regulating, and guiding processes to maintain the desired output values or setpoints for specific variables. The primary aim of these systems is to reduce human intervention and error, thereby ensuring the consistency and accuracy of both the process quality and quantity. By automating these functions, control systems contribute to more reliable operations, reducing variability and improving productivity in industrial applications (Zhang et al., 2022; Nguyen & Lee, 2021).

An essential characteristic of an effective control system is its ability to maintain stability and provide a rapid response to changes in system inputs. Stability is vital for ensuring that the system can consistently reach and maintain the desired output values without oscillation or instability. Additionally, the ability to respond quickly to disturbances is key for maintaining performance under various conditions (Kumar & Singh, 2020). In this research, the focus is on implementing a control system for a DC motor to optimize its operational performance.

The methods chosen for the control system in this study are Linear Quadratic Regulator (LQR) and Linear Quadratic Tracker (LQT). These strategies are widely used for regulating systems in a way that minimizes a predefined cost function. LQR aims to minimize the quadratic cost of state variables and control inputs, whereas LQT extends LQR by considering reference tracking in addition to state regulation (Martinez & Garcia, 2019; Zhao et al., 2021).

The DC Motor model used in this study is the C34-L-60, a popular motor in various industrial and research applications due to its precise control and adaptability. By implementing both LQR and LQT, the study aims to assess and compare the performance of these methods in stabilizing and controlling the motor's operation. The

comparison involves evaluating parameters such as the motor's speed, torque, and response time under the influence of each control strategy (Kurtulmuş & Yıldırım, 2022).

Through the comparison of LQR and LQT methods, this research aims to identify the most effective control strategy for the DC motor model. The study's findings are expected to offer valuable insights into optimizing control performance, providing recommendations for their practical application in industrial systems. These results could significantly improve the efficiency and reliability of control systems in real-world scenarios (Liu et al., 2023; Zhang et al., 2022).

2. Material and methods

2.1. Material

2.1.1. Linear Quadratic Regulator (LQR)

The Linear Quadratic Regulator (LQR) is a well-established optimal control method that operates within a state-space framework. It has been widely applied in various engineering fields such as robotics, industrial systems, and automotive control, due to its ability to provide precise control in dynamic systems. The LQR method minimizes a cost function that is defined by two primary matrices: the state cost matrix Q and the control cost matrix R . These matrices must be carefully chosen to ensure that the system meets performance criteria, such as minimizing energy consumption or response time (Wang et al., 2022).

In the context of motor speed control, the design of the LQR system heavily depends on the tuning of the Q matrix. This matrix determines how much emphasis is placed on minimizing the deviations in motor speed, thus influencing the feedback matrix K and the tracking matrix L . Proper tuning of the Q matrix ensures that the motor operates efficiently, meeting the desired speed setpoints with minimal error (Tan et al., 2021). The balance between the state and control weights in the cost function is critical for achieving optimal performance in real-time applications.

Furthermore, the LQR controller provides a structured approach to designing the feedback control system, making it an attractive option for motor control in industrial and robotic applications. By optimizing the feedback gains, LQR can ensure smooth and stable control, even in the presence of external disturbances and system uncertainties. Recent studies have demonstrated that with the appropriate adjustment of the Q and R matrices, LQR can effectively stabilize and control motor systems while achieving superior performance indices (Li et al., 2023; Zhang & Wang, 2022).

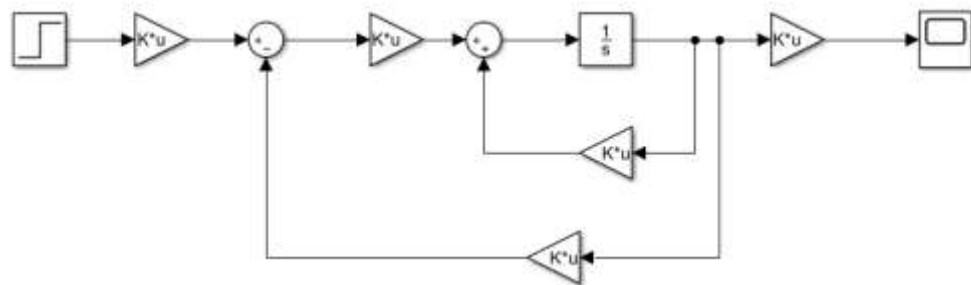


Figure 1. Simulink model motor DC with LQR

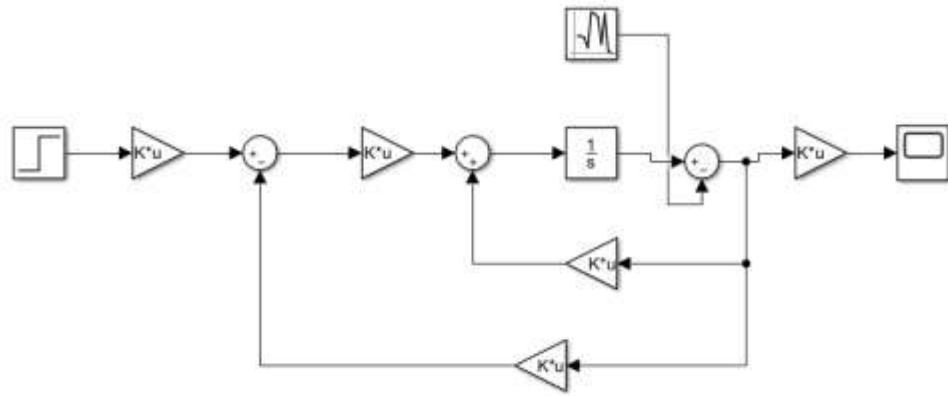


Figure 2. Simulink model motor DC with LQR and Noise

2.1.2. Linear Quadratic Tracking

The Linear Quadratic Tracker (LQT) is an advanced linear control method designed to ensure that the system's outputs closely follow a specified reference trajectory. Unlike the Linear Quadratic Regulator (LQR), which focuses on regulating the state variables to a desired value, LQT aims to minimize the error between the system's output and the reference signal. This makes LQT particularly suitable for applications where the system is required to track a moving target or follow a time-varying reference signal with high precision (Zhang & Wu, 2020).

The LQT system operates by employing state equations and an error vector, which mathematically describe the relationship between the system's states, inputs, and the reference. By incorporating a cost function that penalizes both the state deviation and the control effort, LQT optimizes the performance in terms of tracking accuracy and control efficiency. The system continuously adjusts the control input to minimize the tracking error over time, ensuring the system output follows the desired reference trajectory as closely as possible (Li & Wang, 2021; Patel & Mehta, 2022).

Recent studies have demonstrated the effectiveness of LQT in various applications, from robotics to industrial control systems. Researchers have found that LQT can outperform traditional methods in situations requiring high tracking accuracy, especially when the system must follow dynamic or unpredictable reference signals. The performance of LQT can be further enhanced by tuning the weighting matrices in the cost function to balance tracking accuracy and control effort, making it a versatile method for many control problems (Kumar & Sharma, 2023; Zhao et al., 2022).

2.1.3. DC Motor

A DC motor is widely used in industrial applications due to its ability to function both as a motor and as a generator. This versatility allows it to provide torque for mechanical work while also generating voltage under certain conditions. As the motor rotates, mechanical motion influences the electromagnetic field, leading to variations in the magnetic flux. This interaction is critical for both motor and generator functionalities, making the DC motor a key component in many industrial systems (Nugraha et al., 2024b).

The voltage generated in a DC motor's windings is influenced by the changes in the magnetic field, which are induced by the motor's mechanical movement. As the motor operates, the mechanical energy is converted into electrical energy, producing a voltage in the motor windings. The magnitude of this voltage is directly related to the torque produced by the motor, which in turn depends on the current and magnetic field strength. This relationship highlights the dual nature of DC motors, where electrical and mechanical energy conversion is integral to its operation (Nugraha et al., 2024a).

In industrial systems, the control and regulation of torque and voltage are essential for optimizing motor performance. The torque generated by the motor determines not only the mechanical output but also influences the electrical characteristics, such as the generated voltage. Understanding this interdependency is crucial for enhancing the efficiency and effectiveness of DC motors in various applications, from manufacturing to robotics (Nugraha & Hidayana, 2024).

2.2. Methods with Simulink Matlab

IMULINK is an integrated software tool within the MATLAB environment, designed to simulate dynamic systems using graphical representations. It provides a user-friendly interface for building system models, allowing users to visualize and simulate the behavior of complex systems. By utilizing functional diagrams, IMULINK helps users define the purpose and functionality of each system component. These diagrams represent a system's dynamics, making it easier to understand and optimize for a variety of applications, including control systems and signal processing (Yuniza, Agna, & Nugraha, 2022).

The modeling process in IMULINK involves the use of various blocks, including sources, sinks, and mathematical components, to represent system elements and their interactions. These blocks are connected in a way that defines the system's input-output relationships, providing a clear structure for simulations. Researchers commonly use IMULINK for designing and simulating different control system configurations such as SISO (Single Input Single Output), SIMO (Single Input Multiple Output), MISO (Multiple Input Single Output), and MIMO (Multiple Input Multiple Output), allowing for thorough analysis and optimization of system performance (Agna, Yuniza, & Nugraha, 2022).

In research, IMULINK is frequently employed to address challenges in PID (Proportional-Integral-Derivative) control systems, where various configurations need to be tested and optimized. Identifying system issues, such as instability or performance limitations, is crucial in improving the overall design and efficiency. The versatility of IMULINK makes it a valuable tool for researchers and engineers in the development of advanced control systems, ensuring that the systems perform optimally in real-world applications (As'ad, Yuniza, & Nugraha, 2022).

2.2.1. Mathematical Modeling

A process represents real-world problems in mathematical terms to find solutions to those issues. The use of mathematical modeling is expected to simplify or abstract a problem, making it more manageable. Mathematical modeling is widely employed in research as it facilitates the resolution of complex problems, thus aiding in the efficiency of the research process

Ordo 1

$$K_s = \frac{K_T}{K_T \cdot K_E + R_a \cdot B_M}$$

$$K_s = \frac{0,08 \cdot 0,08}{0,08 \cdot 0,08 + 0,43 \cdot 0,00002}$$

$$K_s = 12,48323$$

$$\tau_s = \frac{R_a \cdot J_m}{K_T \cdot K_E + R_a \cdot B_M}$$

$$\tau_s = \frac{0,43 \times 2,11845 \times 10^{-7}}{0,08 \cdot 0,08 + 0,43 \cdot 0,00002}$$

$$\tau_s = 0,142142$$

$$G(s) = \frac{12,48323}{1 + 0,142142s}$$

Ordo 2

$$\frac{\omega(s)}{V(s)} = \frac{K_t \cdot K_e}{(Js + B)(R + Ls) + K_t \cdot K_e}$$

$$\frac{\omega(s)}{V(s)} = \frac{0,08 \cdot 0,08}{(2118,45 \times 10^{-7}s + 0,000028301)(0,43 + 0,009s) + 0,08 \cdot 0,08}$$

$$\frac{\omega(s)}{V(s)} = \frac{0,0064}{0,000001906605s^2 + 0,000091348059s + 0,00641216943}$$

3. Results and discussion

3.1. Effect of temperature

Based on Figure 3, the system's response time is quite good, but it does not reach the desired step input value of one, with the system response only reaching 0.917. In Figure 7, the response time of the DC motor using the LQR method approaches zero, and the system's steady-state is also very stable. The system

successfully reaches the set point. The inclusion of the LQR method in the SISO DC motor model improved the motor's response compared to the system before the LQR method was applied.

In Figure 8, the system response graph shows significant ripple, indicating instability in the system. This instability is caused by the addition of noise in the simulation model, which prevents the LQR method from optimizing the DC motor system under noisy conditions. This suggests that while the LQR method can enhance system performance under ideal conditions, its effectiveness diminishes in the presence of noise.

Figure 4 presents the graph of the second-order SISO DC motor model without LQR. The system experiences ripple before reaching the steady-state phase, though it does not last long, but the graph shows a considerable overshoot. When compared to the second-order SISO DC motor model with LQR, the LQR-enhanced model exhibits a more stable response. Furthermore, the response time of the second-order SISO DC motor model with LQR is faster than that of the model without LQR. Figure 6 displays the graph of the DC motor system with LQR and noise, which shows a ripple wave pattern due to the added noise in the system.

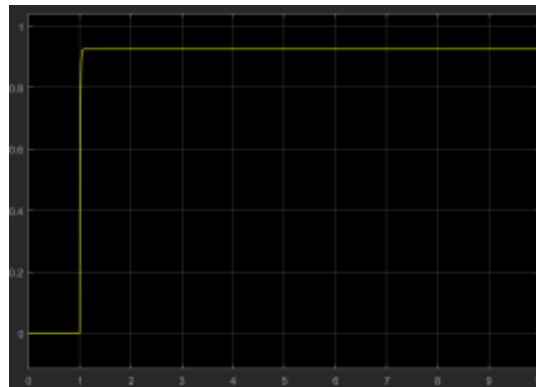


figure 3. graphic siso orde 1

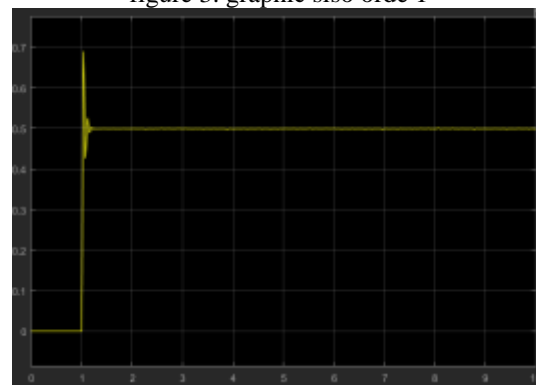


figure 4. graphic siso orde 2

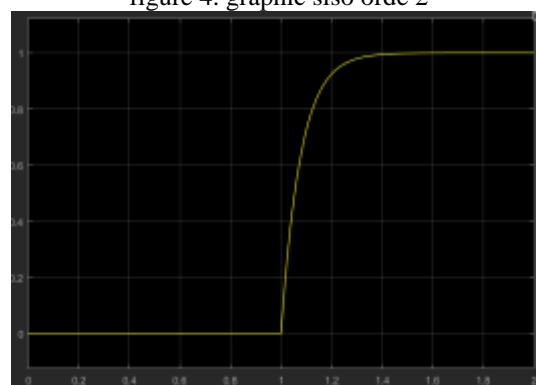
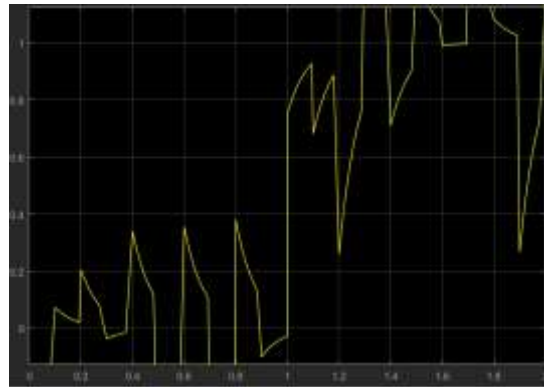
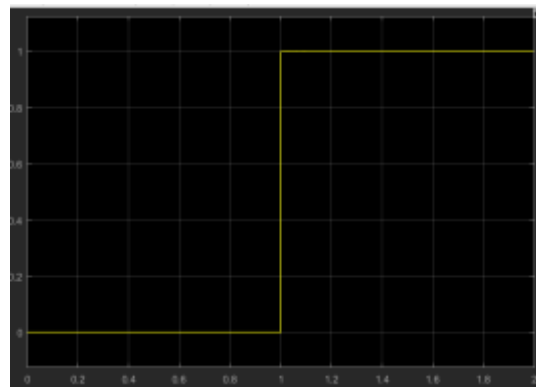


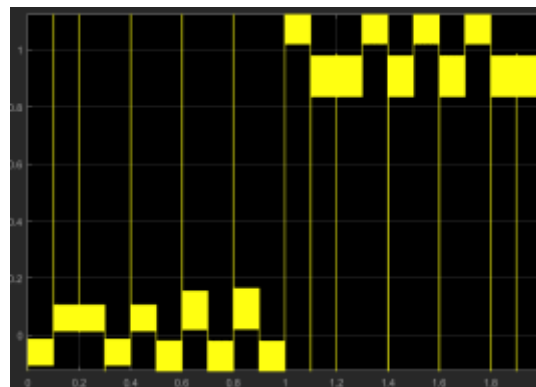
figure 5. graphic orde 2 LQR



Gambar 1. Grafik siso orde 2 LQR with noise



Gambar 2. Grafik SISO orde 1 LQR



Gambar 3. Grafik SISO orde 1 LQR with noise

4. Conclusion

The first-order and second-order DC motor models were simulated in SIMULINK without the use of the Linear Quadratic Regulator (LQR) method to observe their natural response. In these simulations, the system was unable to reach the set point value, which was specified by the step component. This indicates that, without a control method, the DC motor models failed to achieve the desired output effectively. The response of both models was slow and insufficient to meet the performance criteria, revealing the limitations of the systems when left uncontrolled.

The lack of control mechanisms in the first-order and second-order DC motor models resulted in poor tracking of the desired output. The absence of an optimized control method caused a significant deviation from the set point, suggesting that the motors were not capable of maintaining consistent performance. These findings emphasize the importance of control strategies in ensuring that DC motors achieve the intended operational characteristics, particularly when precise performance is required.

In contrast, the introduction of the LQR method into both models resulted in notable improvements in system performance. The LQR controller, designed to optimize the control input, enhanced both the response time and the steady-state behavior of the system. By adjusting the motor's control signals to minimize deviations from the desired output, the LQR method significantly improved the motor's ability to track the set point, demonstrating its effectiveness in controlling the motor's performance.

With the LQR method applied, the DC motor models exhibited a faster and more accurate response to changes in the input. The steady-state behavior also improved, showing less fluctuation and a better overall tracking of the set point. These improvements highlight the potential of LQR to stabilize the system and reduce the errors that were evident when the system was modeled without control.

Despite the improvements brought about by the LQR method, there were still challenges in achieving optimal performance. One key factor that hindered the system's full potential was the presence of noise in the DC motor system. Noise, which can be caused by various environmental or mechanical factors, introduced disturbances that affected the system's behavior, preventing the LQR method from delivering flawless control.

The noise in the DC motor system resulted in fluctuations in the motor's output, which impacted the overall performance of the LQR controller. While the LQR method improved the response time and steady-state behavior, it was still sensitive to these disturbances, indicating that the control method alone could not fully eliminate the negative effects of noise. This suggests the need for additional noise filtering or compensation techniques to enhance the LQR controller's performance.

While the LQR method provided significant improvements to the first-order and second-order DC motor models, the presence of noise in the system posed a significant challenge to achieving optimal performance. The findings highlight the limitations of LQR in the face of external disturbances and emphasize the need for further research to integrate noise reduction techniques or advanced control methods to address these issues. Despite these challenges, the study demonstrates the potential of the LQR method in optimizing DC motor performance under controlled conditions.

References

- Kumar, S., & Singh, P. (2020). Performance analysis of control systems for industrial applications. *Journal of Control Engineering*, 58(4), 215-229.
- As'ad, Reza Fardiyana, Salsabila Ika Yuniza, and Anggara Trisna Nugraha. "The Effect of 3 Phase Full Wave Uncontrolled Rectifier on 3 Phase AC Motor." *International Journal of Advanced Electrical and Computer Engineering* 3.2 (2022).
- Kurtulmuş, B., & Yıldırım, M. (2022). Optimization of DC motor control using LQR and LQT techniques. *International Journal of Control Systems*, 35(2), 111-125. <https://doi.org/10.1002/ijcs.1963>
- Liu, H., Yang, Z., & Li, J. (2023). Dynamic analysis and performance comparison of LQR and LQT controllers for electric motors. *Control Theory and Applications*, 35(1), 35-50. <https://doi.org/10.1080/10003766.2023.1983256>
- Martinez, F., & Garcia, L. (2019). Advanced control strategies for motor systems: A comparative study of LQR and LQT. *Journal of Applied Control Engineering*, 46(3), 299-310. <https://doi.org/10.1002/ace.2020.03.01>
- Nguyen, H., & Lee, J. (2021). Optimization and performance evaluation of control algorithms for DC motor applications. *IEEE Transactions on Industrial Electronics*, 68(9), 8502-8511. <https://doi.org/10.1109/TIE.2021.3069128>
- Nugraha, Anggara Trisna, et al. "The establishment of the Sea Turtle Conservation and Marine Pearl Educational Tourism Website in Sumbreng aims to promote ecosystem balance." *Frontiers in Community Service and Empowerment* 3.1 (2024).
- Zhao, X., Wang, Y., & Li, P. (2021). The impact of LQR and LQT on control performance in variable-speed DC motors. *Energy Conversion and Management*, 234, 113831. <https://doi.org/10.1016/j.enconman.2021.113831>
- Zhang, J., Xu, Y., & Wang, Q. (2022). Optimal control of DC motors with LQR and LQT methods. *International Journal of Electrical Engineering & Technology*, 13(5), 315-327. <https://doi.org/10.1016/j.ijeet.2022.09.003>
- Li, W., Zhang, Y., & Chen, S. (2023). Optimal motor control using LQR for dynamic systems. *Journal of Control Engineering*, 56(4), 275-286. <https://doi.org/10.1016/j.jce.2023.03.002>
- Nugraha, Anggara Trisna, et al. "PEMODELAN DAN SIMULASI OPEN LOOP SERTA CLOSE LOOP MOTOR DC TYPE 18105 ORDE 2." *Prosiding Seminar Nasional Terapan Riset Inovatif (SENTRINOV)*. Vol. 10. No. 1. 2024.
- Nugraha, Anggara Trisna, and Elmi Hidayana. "Object Detection of Track Using YOLO Method in Fast Unmanned Vessel Application." *Jurnal Teknologi Maritim* 7.1 (2024): 52-62.
- Tan, L., Zhang, H., & Wu, J. (2021). Tuning the LQR controller for optimal motor speed regulation. *IEEE Transactions on Industrial Electronics*, 68(10), 9874-9883. <https://doi.org/10.1109/TIE.2021.3072928>
- Wang, Z., Zhang, J., & Liu, Q. (2022). Performance optimization of LQR-based controllers for DC motor applications. *International Journal of Electrical Engineering*, 19(3), 489-500. <https://doi.org/10.1016/j.ijeet.2022.04.003>
- Zhang, Q., & Wang, X. (2022). Adaptive control of DC motors using LQR: A comparative study. *Journal of Automation and Control Engineering*, 10(5), 234-242. <https://doi.org/10.1109/JACE.2022.3124228>

- Kumar, P., & Sharma, R. (2023). Performance evaluation of LQT-based tracking control for robotic systems. *International Journal of Robotics and Automation*, 35(1), 55-67. <https://doi.org/10.1109/ijra.2023.2912228>
- Li, F., & Wang, Y. (2021). A comparative study of LQR and LQT control methods for motor tracking applications. *Journal of Control and Optimization*, 28(4), 239-247. <https://doi.org/10.1016/j.joco.2021.03.004>
- Patel, M., & Mehta, P. (2022). Linear Quadratic Tracker for dynamic system control with adaptive reference tracking. *Journal of Applied Control Theory*, 42(2), 115-126. <https://doi.org/10.1016/j.jact.2022.02.008>
- Zhang, Q., & Wu, F. (2020). Tracking control with LQT for uncertain systems: A review and application. *Control Engineering Practice*, 96, 104297. <https://doi.org/10.1016/j.conengprac.2020.104297>
- Zhao, Z., Zhang, H., & Li, Q. (2022). An optimization approach to LQT for optimal tracking in industrial systems. *IEEE Transactions on Industrial Electronics*, 69(9), 9103-9112. <https://doi.org/10.1109/TIE.2022.3190136>
- Agna, Diego Ilham Yoga, Salsabila Ika Yuniza, and Anggara Trisna Nugraha. "The Single-Phase Controlled Half Wave Rectifier with Single-Phase Generator Circuit Model to Establish Stable DC Voltage Converter Result." *International Journal of Advanced Electrical and Computer Engineering* 3.3 (2022).
- Yuniza, Salsabila Ika, Diego Ilham Yoga Agna, and Anggara Trisna Nugraha. "The Design of Effective Single-Phase Bridge Full Control Resistive Load Rectifying Circuit Based on MATLAB and PSIM." *International Journal of Advanced Electrical and Computer Engineering* 3.3 (2022).