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Comparative Analysis of PID and LQR Controllers for Speed Regulation of Series DC Motors

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Abstract

A DC motor is a widely used electromechanical device known for its ease of application and versatile speed regulation capabilities, making it essential in various industries, robotics, and household appliances. Among different types of DC motors, the series DC motor is noted for its high starting torque, which can cause significant overshoot at startup. Moreover, this motor exhibits inherent instability, with speed decreasing at higher torques and increasing under low loads, potentially reaching very high speeds in no-load conditions. In order to achieve precise speed control and mitigate overshoot, the implementation of an effective control system is crucial. This study presents a comparative simulation analysis, conducted using MATLAB, between two widely used controllers: PID (Proportional-Integral-Derivative) and LQR (Linear Quadratic Regulator), for regulating the speed of a series DC motor. The results demonstrate that both controllers achieve minimal errors, with the PID controller delivering a faster rotor speed response compared to the LQR controller. However, the PID controller exhibits a notable overshoot of approximately 20%, while the LQR controller successfully eliminates any overshoot. Additionally, the initial current surge observed with the PID controller is significantly higher than with the LQR controller, with the PID's starting current overshoot reaching about 460%, compared to only 188% for the LQR.

Keywords: Dc motor, series, LQR, PID, speed control, matlab

1. Introduction

DC motors are widely utilized in various fields, including industrial applications, robotics, and household appliances, due to their versatile functionality and ability to regulate speed over a broad range (Saturn, 2000). One specific type of DC motor, the series DC motor, is well-known for its large starting torque, which can be both an advantage and a challenge (Lister, 1986). The motor's large starting torque often leads to significant overshoot during initial startup, and it can cause the motor to be unstable in operation. Under high load, the motor's speed tends to decrease, and conversely, when the motor is unloaded, its speed can increase drastically, often reaching dangerously high levels (Rijono, 1997).

In many practical applications, a motor's speed regulation is crucial, and this is especially important to ensure that the motor performs smoothly and without excessive vibration or mechanical shock, particularly during the startup phase (Chapman, 2005). To mitigate such challenges and achieve reliable and controlled speed performance, a robust control system is essential (Nugraha et al., 2022). Control systems are primarily used to address issues like overshoot, settling time, and overall system stability when a motor is transitioning to a steady state (Dwivedi & Dohare, 2015).

A widely adopted control technique is the Proportional-Integral-Derivative (PID) controller, which is favored for its simple structure, ease of implementation, and straightforward parameter tuning (Kuo, 1995). Despite its advantages, the PID controller has limitations in certain applications, especially in situations requiring more optimal performance. Therefore, alternative control methods, such as the Linear Quadratic Regulator (LQR), have been explored. The LQR offers potential benefits in terms of achieving superior system response and stability (Philips & Harbor, n.d.).

This study aims to compare the performance of both PID and LQR controllers in controlling a series DC motor. The research will investigate how to determine the optimal controller parameters for both PID and LQR methods, how to design and simulate each controller to achieve stable motor operation at the desired speed, and how to compare the system responses of the two controllers. Specifically, this study seeks to analyze the effectiveness of each controller in driving the motor to a steady state with the desired speed, to evaluate the optimal performance of both controllers based on the simulation results, and to generate comparison curves illustrating the system responses of PID and LQR controllers.

2. Material and methods

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2.1. Proportional Integral Derivative

Control Proportional Integral Derivative Control (PID) is a feedback mechanism controller that is usually used in industrial control systems (Lewis, 1996) (Ogata, 1997). A PID controller continuously calculates the error value as the difference between the desired setpoint and the measured process variable (Ogata, 2010). The controller attempts to minimize the error value over time by setting a control variable, such as the position of the control valve, damper, or power on the heating element, to a new value determined by the sum. It can combine proportional, integral, and derivative controllers (Bimbra, 1990). This controller is represented by the following equation

$$m(t)=Kp.e(t)+\frac{Kp}{Ti}\int_0^t e(t)dt+Kp.Td\frac{de(t)}{dt}$$

where *Kp* is a proportional constant, *Ti* is the integral time, and *Td* is the derivative time (Linsley, 1998). Equation 1 is an equation in the time domain. To facilitate writing in the program, equation 1 is converted into discrete form, using *finite differential* which is presented in the following equation:

$$\frac{Df}{Dt}||_k=\frac{(f_k-f_{k-1})}{\Delta t}$$

$$\int e(t)dt=\sum_{k=0}^{n} e_k.\Delta t$$

So equation 1 becomes:

$$m_n=K_p\left[T_d\frac{(e_n-e_{n-1})}{\Delta t}+e_n+\frac{1}{T_i}\sum_{k=0}^{n} e_k.\Delta t\right]$$

Where:

$$K_i=K_p\frac{T_s}{T_i} \text{ and } K_d=K_p\frac{T_d}{T_s}$$

$$\text{with } \Delta t=T_s \text{ if } S_n=S_{n-1}+e_n$$

Then the PID controller equation in discrete form is as follows:

$$m_n=K_p.e_n+K_i.S_n+K_d.(e_n-e_{n-1})$$

Where S_n = number of errors
S_{n-1} = number of previous errors
e_n = current errors
e_{n-1} = previous errors
m_n = current output.

equation *transfer function* of a DC motor is as follows:

$$\frac{Kt}{(L\times J)s^2+(L\times b+R\times J)s+(R\times b+Kt\times Ke)}$$

2.2. Linear Quadratic Regulator Control

To get the desired performance criteria that meet the physical limits is the goal in optimal control (Anggono, 2011). Regulator problems will be solved by using the optimal control method on a linear system with quadratic criteria (Fitzgerald, 1992). It is said to be linear because the model and controller form is linear, while quadratic because it has a cost function that is quadratic and because the reference system is not a function of time, it is called a regulator (Berahim, 1994).

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The results of linearization of a linear plant are obtained in the form:

$$x' = Ax + Bu$$
$$y = Cx$$

Where A = Matrix system
B = Matrix input
C = Matrix output
y = state output
x = state system
u = state input.

Determination of the matrix values of Q and R is a value that will be determined first in the design of the optimal LQR controller. When the Q and R matrices are obtained, the next step is to determine the system performance index. The use of the performance index is determined according to the criteria of the matrix prices Q and R [2].

$$J(t_0) = \frac{1}{2} x^2(T) S(T) x(T) + \frac{1}{2} \int_{t0}^{T} (x^2 Qx + u^2 Ru)$$

With the provision of:

$$S(T) \geq 0, Q \geq 0, R > 0$$

Where t_0 = initial time
= = end time
$x(\tau)$ final state matrix
Q = positive semi definite matrix
R = positive definite matrix
S = positive semi definite matrix.

2.3. MATLAB (Matrix Laboratory)

Matrix Laboratory (MATLAB) software is a program for analyzing and computing numerical data, it is also an advanced mathematical programming language, built on the premise of using the properties and forms of matrices (Mehta & Chiasson, 1998) (Nugraha, Priyambodo, & Sarena, 2022). MATLAB is extensible, meaning that users can write new functions to add to the library when the available built-in functions cannot perform certain tasks (Dubey & Srivastava, 2013). The programming skills required are not too difficult if we already have experience in programming other languages such as C, PASCAL, or FORTRAN (Ravi, Widodo, & Nugraha, 2021).

2.4. Methods

A. Data Parameter

Table 1. Parameter

Parameter	Symbol	Large and
Moment of Inertia	J_m	0.0007046 kg.m^2
Friction coefficient	B_m	0.0004 Nm/(rad/s)
Torque constant	K_t	0.1236 Nm/A Back tension
constant	K_b	0.1235 V/(rad/s)
Total resistance of coil	R_t	7.2 ohm
Total inductance of coil	L_t	0.0917 H

B. Obtaining PID Parameter

Values that need to be found from the curve are the time delay value (L) and the time constant value (T). By using the straight-line equation, the values of L and T will be determined. In Figure 4.4 it can be seen that there are 2 points with coordinates: X1 = 0.03688, Y1 = 0.5884 and X2 = 0.3227, Y2 = 6.791

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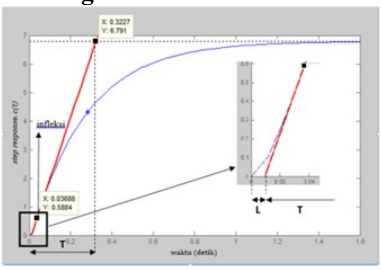
Figure 1. PID Parameter



The general form of the straight-line equation:

$$Y_2 - Y_1 = m(X_2 - X_1)$$

where *m* is the slope of the line.

$$6.791 - 0.5884 = m(0.3227 - 0.03688)$$
$$m = \frac{6.791 - 0.5884}{0.3227 - 0.03688}$$
$$m = 21.701$$

The tangent line touches the x-axis at a point with coordinates (X,0), then

$$Y_2 - 0 = m(X_2 - X)$$
$$6.791 - 0 = 21.701(0.3227 - X)$$
$$X = 0.3227 - \frac{6.791}{21.701}$$
$$X = 0.009765$$

From Figure 4.4 it can be seen that the value of L is equal to X. Thus T equal to $X_2 - L$

$$L = 0.009765$$

T = 0.3227 – 0.009765 = 0.3130.

After the L and T values are obtained, we can determine the PID parameter value
Proportional Constant (K_p) :

$$K_p = 1.2\left(\frac{T}{L}\right)$$
$$K_p = 1.2\left(\frac{0.3130}{0.009765}\right)$$
$$K_p = 38.464$$

Integral Constant (K_i) :

$$T_i = 2L, K_i = \frac{K_p}{T_i} \text{ then:}$$
$$K_i = \frac{K_p}{2L} = \frac{38.464}{2(0.009765)} = 1969.483$$

Derivative Constants (K_d) :

$$T_i = 0.5L$$
$$K_d = T_d$$
$$K_p \text{ then:}$$
$$K_d = 0.5L(K_p) = 0.5(0.009765)(38.464) = 0.1878$$

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C. LQR Parameter

In the LQR simulation, the motor is modeled in the form of a state space, namely :

$\dot{X}(t) = Ax(t) + Bu¿)$

Y(t) = Cx(t)

Where matrix A, B, C is determined by :

$$A = \begin{bmatrix} \frac{-R}{L} - \frac{K_b}{L} \frac{K_t}{J_m} - \frac{B_m}{J_m} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L} 0 \end{bmatrix}, \text{and } C = \begin{bmatrix} 0 1 \end{bmatrix}$$

By entering the data from Table 4.1 into the matrix equation, we get:

$$A = \begin{bmatrix} \frac{-7.2}{0.0917} - \frac{0.1236}{0.0917} \frac{0.1236}{0.0007046} - \frac{0.0004}{0.0007046} \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{0.0917} 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 1 \end{bmatrix}, \text{and } D = \begin{bmatrix} 0 \end{bmatrix}$$

Obtaining LQR Parameters To obtain the Q and R matrices, the matlab script program using the trial and error method can be seen in Appendix II, where the conditions for the Q matrix are real positive semidefinite matrices (Q 0) and the R matrix is the matrix real positive definite (R > 0).

We set the initial value of Q = $\begin{bmatrix} 1 0 0 1 \end{bmatrix}$ and

R = $\begin{bmatrix} 1 \end{bmatrix}$,

K=[1.2892 0.6016]

D. Circuit

- **PID**

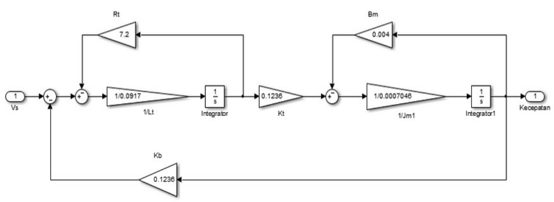

Figure 2. *3.Series DC Motor Simulink Circuit with PID*

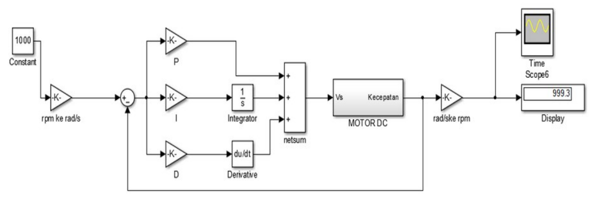

Figure 3. PID Simulation

- **LQR**

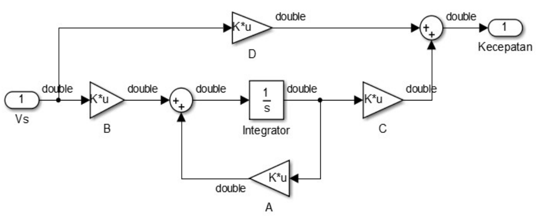

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Figure 4. Series DC Motor Simulink Circuit with LQR

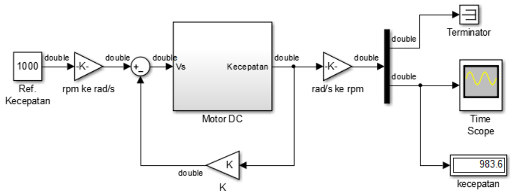

Figure 5. Simulation Circuit LQR

3. Results and discussion

3.1. PID Simulation

A. PID Simulation at 1300 rpm Reference Speed

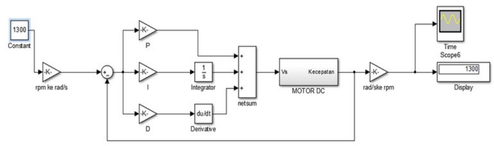

Figure 5. PID Simulation Circuit at 1300 rpm Reference Speed A

steady speed of 1300 rpm was obtained. Rotor speed response at a reference speed of 1300 rpm is shown in Figure 6.

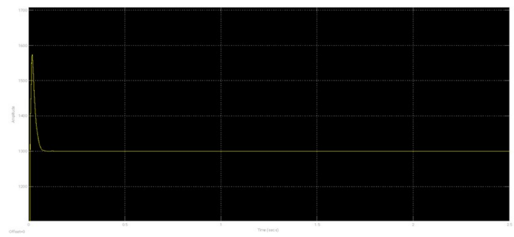

Figure 6. Rotor speed response at a reference speed of 1300 rpm with PID control

Parameters obtained from the rotor speed response:
Rise time : 6,995 ms
Settling time : 53.7 ms
Max. Overshoot : 21.09 %
Steady state error : 0 %

B. PID Simulation at 1600 rpm Reference Speed

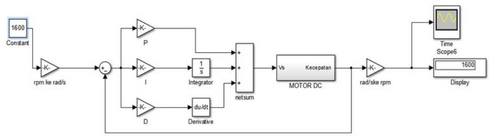

Figure 7. PID Simulation Circuit at 1600 rpm Reference Speed

Obtained steady speed of 1600 rpm. The rotor speed response at a reference speed of 1600 rpm is shown in Figure 8.

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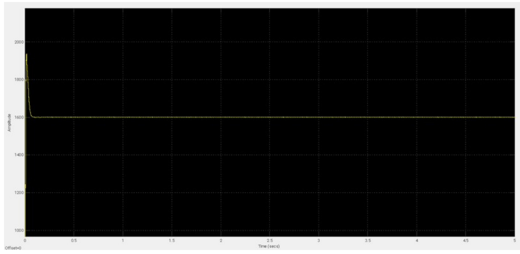

Figure 8. .Rotor speed response at a reference speed of 1600 rpm with PID Control

The obtained rotor speed response parameters are :
Rise time : 7.067 ms
Settling time : 53.8 ms
Max. Overshoot : 21.05%
Error steady state : 0 %

3.2. LQR Simulation

A. LQR simulation at a reference speed of 1300 rpm

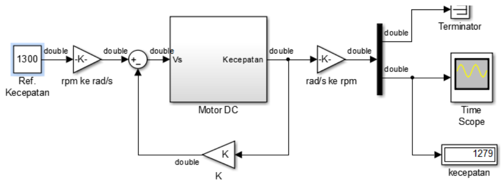

Figure 9. LQR simulation circuit at a reference

speed of 1300 rpm A steady speed of 1300 rpm was obtained. Rotor speed response at a reference speed of 1300 rpm is shown in Figure 10.

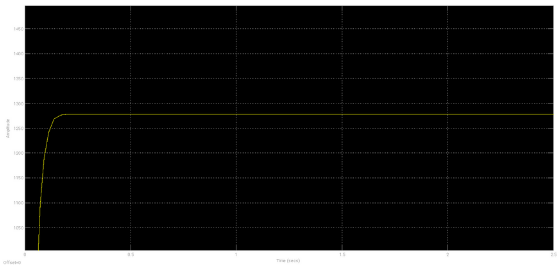

Figure 10. Rotor speed response at a reference speed of 1300 rpm with LQR control

Parameters obtained from the rotor speed response:
Rise time : 89.743 ms
Settling time : 166.9 ms
Max. Overshoot : 0 %
Steady state error : 0 %

B. LQR Simulation at 1600 rpm Reference Speed

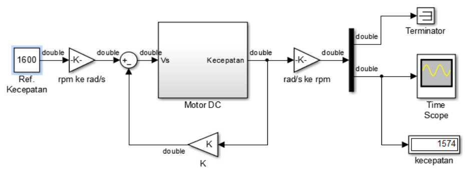

Figure 11. LQR Simulation Circuit at 1600 rpm Reference Speed

Obtained steady speed of 1600 rpm. The rotor speed response at a reference speed of 1600 rpm is shown in Figure 12.

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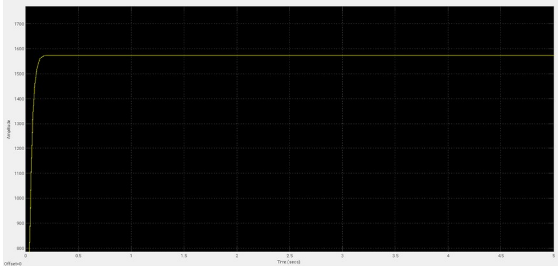

Figure 12. Rotor speed response at a reference speed of 1600 rpm with LQR control

Parameters of the rotor speed response obtained:
Rise time : 90.340 ms
Settling time : 164.1 ms
Max. Overshoot : 0 %
Error steady state : 0 %

C. Comparison Table

Table 2. Result comparison

Controller	*Speed (rpm)*	*Rise Time (ms)*	*Settling Time (ms)*	*Max. Over Shoot (%)*	*Steady Error (%)*
PID	1300	6,995	53.7	21.09	0
	1600	7.067	53.8	21.05	0
LQR	1300	89.743	166.9	0	0
	1600	90.340	164.1	0	0

4. Conclusion

From the simulation results, several conclusions can be drawn, namely as follows:

1. In achieving steady speed, PID provides a shorter time than LQR as we can see in the results simulation, rise time and settling time obtained using PID is smaller than using LQR.
2. The rotor speed response characteristic obtained by using LQR has no overshoot at all, while using PID the resulting overshoot is quite large, which is around 20%.
3. Of the five speed variations experiments for each controller, PID has a steady state error twice while LQR has a steady state error once.
4. The simulation results show that the speed variation applied to a series dc motor with PID and LQR control does not significantly affect the response rotor speed to reach steady speed.
5. Percentage max. The armature current overshoot that occurs using the PID controller is around 460% while using the LQR controller is around 188%, which means that the starting current using the PID controller is much larger than using the LQR controller.

Credit authorship contribution statement
Author Name: Conceptualization, Writing – review & editing. **Author Name**: Supervision, Writing – review & editing. **Author Name**: Conceptualization, Supervision, Writing – review & editing.

References

Saturn. 2000. Basic Electrical Engineering and Power Electronics. Jakarta: Gramedia Pustaka Utama.
Lister, Eugene C. 1986. Machinery and Electrical Circuits (6th Edition). Jakarta: Erlangga Publisher.
Rijono, Yon. 1997. Basic Electrical Power Engineering. Yogyakarta: Andi Offset.
Chapman, Stephen J. 2005. Electric Machinary Fundamentals 4th Edition. Singapore: McGraw-Hill International Edition.
Dwivedi, Rajkumar and Devendra Dohare. 2015. PID Conventional Controller and LQR Optimal controller for Speed analysis of DC Motor: A Comparative Study. International Research Journal of Engineering and Technology. 02(08): 508-511.
C. Kuo, Benjamin. 1995. Automatic Controls System Seventh Edition. New Jersey: Prentice Hall Inc.

DOI xxxxxx

L. Philips, Charles & Royce D. Harbor. Feedback Control Systems 3e. New Jersey: Prentice Hall Inc.
Lewis, FL 1996. Optimal Control. Canada: John Wiley & Sons Inc.
Ogata, Katsuhiko. 1997. Automatic Control Techniques (System Settings) Volume 1 Second Edition. Jakarta: Erlangga.
Ogata, Katsuhiko. 2010. Modern Control Engineering Fifth Edition. New Jersey (US): Pearson Education Inc.
Bimbra, PS 1990. Electrical Machinery. Delhi: Khana Publisher.
Linsley, Trevor. 1998. Basic Electrical Installation Work Third Edition. Kidlington (UK): Elsevier Ltd.
Anggono, Tri. 2011. Design of a Steam Pressure Control System on a Small-Scale Steam Drum Boiler Using PID and LQR [thesis]. Depok (ID): University of Indonesia.
Fitzgerald. AE 1992. Electrical Machinery (4th Edition). Jakarta: Erlangga Publisher.
Berahim, Hamza. 1994. Introduction to Electrical Engineering. Yogyakarta: Andi Offset.
Mehta, Samir & John Chiasson. 1998. Nonlinear Control of a Series DC Motor: Theory and Experiment. IEEE Transactions on Industrial Electronics. 45(1): 134-141.
Dubey, Saurabh & SK Srivastava. 2013. A PID Controlled Real Time Analysis of DC Motor. International Journal of Innovative Research in Computer and Communication Engineering. 01(8): 1965-1973.
Anggara Trisna, Dadang Priyambodo, and Sryang Tera Sarena. "Design A Battery Charger with Arduino Uno-Based for A Wind Energy Power Plant." JPSE (Journal of Physical Science and Engineering) 7.1 (2022): 23-38.
Ravi, Alwy Muhammad, Hendro Agus Widodo, and Anggara Trisna Nugraha. "PENGARUH PENGGUNAAN METODE KONTROL PI PADA KONTROL EKSITASI GENERATOR SINKRON." Seminar MASTER PPNS. Vol. 6. No. 1. 2021.
Nugraha, Anggara Trisna, et al. "Brake Current Control System Modeling Using Linear Quadratic Regulator (LQR) and Proportional integral derivative (PID)." Indonesian Journal of Electronics, Electromedical Engineering, and Medical Informatics 4.2 (2022): 85-93.