

Design of LQR and LQG Control Systems for the RIG 38-100 Process

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Abstract

This paper presents the design of a control system to regulate water levels and flow rates, ensuring that the water level remains at a normal setpoint despite variations in load or input conditions. This approach aims to enhance the safety and efficiency of water level management. Nonlinear behaviors in the system are addressed during level regulation, but for analytical simplicity, the system is typically assumed to be linear in foundational studies. In this research, the nonlinear system is linearized to facilitate modeling in a state-space representation. The proposed control strategy employs the Linear Quadratic Gaussian (LQG) optimal control method, which integrates the Linear Quadratic Regulator (LQR) technique with the Kalman Filter. The LQR component is designed to optimize the system states, while the Kalman Filter provides noise-free estimations of water level and flow rate. The controller's implementation and performance evaluation were conducted using MATLAB/Simulink software. The study compares the plant's dynamic response under loaded and unloaded conditions, demonstrating the effectiveness of the proposed method in achieving stable and efficient control.

Keywords: Water level, Linier Quadratic Regulator, Filter Kalman.

1. Introduction

In the chemical industry, operational processes aim to manage a series of equipment to ensure that operations run in accordance with the applicable unit operations (Nugraha and Febrianti, n.d.). To achieve this goal, a robust control system is essential. One critical aspect of this control is managing the liquid level within industrial processes (Pambudi et al., 2021). If the liquid level is not properly controlled, the industrial process may be disrupted. Excessive liquid levels can result in overflow, potentially damaging equipment and disrupting operations. Conversely, insufficient liquid levels can prevent the process from functioning as intended. Thus, maintaining precise liquid level control is imperative in industrial applications (Magriza et al., 2021).

One prominent example is the distillation or separation of oil and water in power generation systems, a widely used process (Setiawan et al., 2023). This operation relies on liquid level control to separate the two compounds. In such systems, water level control serves as a boundary between oil and water (Yuniza et al., 2022). When water and oil are stored in a single container, separation occurs naturally, with water occupying the bottom layer and oil floating above it, separated by the water level control boundary. As water continuously enters the container, the oil rises and is eventually discharged through a conduit to a separate reservoir. The water level control system detects the liquid level within the distillation tank (Litwhiler, 2012). When the liquid reaches a specified threshold, water inflow ceases, and the level is adjusted by releasing water until it returns to the desired range. This cyclical operation ensures effective separation (Athans, 1986).

A key challenge in managing liquid levels is the nonlinear behavior observed, where the water flow rate is proportional to the square root of the height of the liquid (Stein and Athans, 1987). This nonlinearity must be addressed by linearizing the system during modeling, which involves deriving the state-space representation of the liquid height for control purposes.

2. Material and methods

2.1. Process Control System

A process control system can be defined as the set of functions and operations necessary to transform materials physically or chemically (Kalbat, 2013). This system is commonly associated with the manufacturing or processing of products in industrial settings. A process control system consists of various devices and electronic equipment designed to ensure stability and accuracy during production processes. Each component within the process control system plays a critical role, regardless of its form or size (Nugraha, 2022).

The variables related to a process, such as flow rate, pressure, fluid level, and temperature, can be classified into two primary groups:

1. Input Variables: These represent the energy quantities originating from the environment or provided externally, which can influence the process dynamics.

2. Output Variables: These are the energy quantities generated by the process that can affect the surrounding environment.

Further classification of input variables includes:

- Manipulated Variables (MV): Input variables whose values can be adjusted by a human operator or a control mechanism.
- Disturbances: Input variables whose values cannot be controlled directly by the operator or control mechanism.

Similarly, output variables can be categorized as follows:

- Measured Output Variables: These outputs can be directly observed and quantified using measurement tools.
- Unmeasured Output Variables: These outputs cannot be directly measured.

In process control systems, output variables are also referred to as process variables. Figure 1 provides a schematic diagram illustrating the different types of variables influencing the process control plant.

The components of a process control system can be divided into several elements:

- Process: The main operation that requires regulation.
- Sensor (Sensing Element): Devices used to detect changes in process variables.
- Transducer: Instruments that convert one form of energy into another for measurement or control purposes.
- Transmitter: Devices that send signals representing process measurements to controllers.
- Transmission Lines: Channels through which signals are transmitted within the system.
- Controller: The central element that processes signals and determines control actions based on the desired setpoints.
- Final Control Element (e.g., Control Valves): Devices that execute control actions to adjust the process.

These elements collectively form a process control plant, as shown in the block diagram of the process control system.

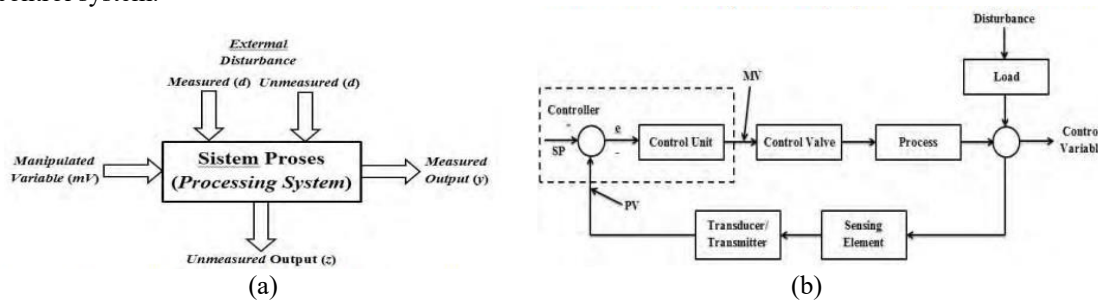


Figure 1. (a) Variabel Input and Output
(b) Block Diagram system

2.2. Plant Level and Flow Process Rig 38-100

The two-tank interconnected system used as the process plant in this study is the Level and Flow Process Rig 38-100, a device specifically designed for laboratory applications in control system theory. It serves as a representative model of an industrial plant system, enabling students and researchers to analyze and implement control strategies effectively.



Figure 2. Level and Flow Process Rig 38-100

The Level and Flow Process Rig 38-100 comprises two tower tanks located above a reservoir designed to store water. Water is pumped from the reservoir into the first tank (on the right). When the water level in the first tank reaches a sufficient height, it flows into the second tank (on the left). The water level in each tank can be observed using a ruler-like scale mounted on the front of the tanks. Each tank is equipped with an outlet pipe

at its base, where water discharge can be controlled via adjustable manual valves (Singh and Saikia, 2016). Variations in valve configurations lead to changes in the system dynamics; thus, valve settings must remain constant during system modeling.

The Level and Flow Process Rig 38-100 consists of several key components, including a servo valve, flowmeter, and manual valve connected to the water pipeline. Additionally, supporting devices include the Process Interface 38-200, Process Controller 38-300, Float Level Transmitter 38-401, Pulse Flow Transmitter 38-421, Digital Display Module 38-490, and a Centrifugal Pump. The primary components of the system are detailed below (Revil et al., 2017) (Zhang et al., 2010):

- **Servo Valve:** The servo valve controls the flow rate of water pumped from the reservoir. It regulates the amount of water entering the upper tanks to maintain the desired water level, aligning with the setpoint values. The servo valve operates vertically and is controlled by an electrical current ranging from 4 mA (fully closed) to 20 mA (fully open).
- **Level Sensor and Flow Sensor:** The Level Sensor and Flow Sensor serve as the primary measurement elements in the system. The level sensor monitors the water height in the upper tank, while the flow sensor measures the water flow rate through the piping system.
- **Manual Valve and Solenoid Valve:** The manual valve and solenoid valve are installed at the outlet of the upper tanks to control water discharge back into the reservoir, simulating load disturbances in the plant.
- **Process Interface 38-200:** The Process Interface 38-200 acts as a power supply hub for sensors, controllers, and actuators. It includes four input terminals and one output terminal, ensuring seamless integration of the control system components and facilitating robust data exchange between elements.
- **Process Controller 38-300:** The Process Controller 38-300, manufactured by ABB, is an industrial-grade controller based on a microprocessor platform. It supports up to four logic inputs and four relay outputs, offering multiple control options, including PID control with up to three configurations. This controller also provides functionality for adjusting setpoints, monitoring responses, and managing parameters directly related to the process.
- **Centrifugal Pump:** The centrifugal pump used in this setup differs from jet pumps by pushing water to the surface rather than pulling it.

The Level and Flow Process Rig 38-100, with its dynamic components and configuration options, provides a robust platform for implementing advanced control strategies such as LQR (Linear Quadratic Regulator) and LQG (Linear Quadratic Gaussian) controllers (Pan et al., 2011). By enabling precise control over water levels and flow rates, this system supports real-time experimentation and validation of theoretical control models, making it an essential tool for engineering research and education (Samakwong and Assawinchaichote, 2016).

This detailed description of the system aligns with the principles of modern engineering research and offers practical insights for inclusion in journals, seminars, and technical proceedings. It ensures clarity and thoroughness as required by reviewers and editors, providing a comprehensive understanding of the RIG 38-100 setup and its role in process control innovation.

2.3. Cascade controller

Cascade controllers are frequently employed in process control systems to enhance performance and ensure greater system stability compared to single-loop controllers (Lobanoff and Ross, 2013). By utilizing a cascade configuration, the control system can achieve improved dynamic response and disturbance rejection (Çelik, 2021). This approach aligns well with the objectives of modern engineering applications, where reliability and efficiency are critical.

The implementation of cascade controllers serves two primary purposes:

- **Eliminating the Impact of Disturbances:** By isolating disturbances in the inner loop, the overall system becomes more robust to external and internal variations, resulting in a more stable process.
- **Enhancing Control Loop Dynamics:** Cascade control systems can respond more rapidly and effectively to changes in setpoints or operating conditions compared to single-loop configurations.

The controller found in the Outerloop, G_{c1} , is called the master or primary controller (main controller) and the one found in the Innerloop, G_{c2} , is called the auxiliary or secondary controller (auxiliary controller) (Mukherjee et al., 2021). From the Block Diagram schematic above, it can be understood that the basic characteristics of cascade control are that the combination of the first two controllers (primary controller) is the setpoint for the next controller (secondary controller).

2.4. Linear Quadratic Regulator (LQR)

The LQR design used in this research is an infinitetime LQR system (Rahman et al., 2017). This method is used in a linear time invariant (LTI) system with the following plants.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ Y &= Cx \end{aligned}$$

Where:

- $x(t) \in \mathbb{R}^n$ = vector state ordo -n,
- $u(t) \in \mathbb{R}^r$ = vector control ordo -r,
- $A \in \mathbb{R}^{n \times n}$ = matrix state,
- $B \in \mathbb{R}^{n \times r}$ = matrix control.

Cos function equation:

$$J = \frac{1}{2} \int_0^{\infty} [x^T Q x(t) + U^T(t) R u(t) dt]$$

Where :

- $Q \in \mathbb{R}^{n \times n}$ = the weighting matrix for the state, and must be positive semidefinite, i.e $Q \geq 0$
- $R \in \mathbb{R}^{r \times r}$ = weighting matrix for control, and must be positive definite, i.e $R > 0$

With these requirements:

- Matrix Q must be positive semidefinite, $x(t)^T Q x(t) \geq 0$ for every vector state $x(t)$
- Matrix R must be positive definite, $u(t)^T R u(t) > 0$ for every vector state $u(t)$

Block digram LQR below:

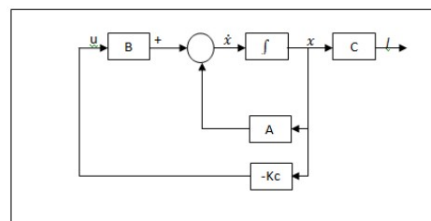


Figure 3. Block diagram LQR

2.5. Kalman Filter

The Kalman Filter is a state estimator widely used in control systems, offering distinct advantages over other filtering techniques (Khodarahmi and Maihami, 2023). Unlike conventional filters, the Kalman Filter can estimate the internal states of a system based on noisy observations of the plant's outputs (Daraz, 2023). This capability makes it particularly effective in processes where measurement noise is present, allowing for the accurate estimation of system states despite imperfect or corrupted data.

The primary strength of the Kalman Filter lies in its ability to reduce or eliminate noise from the measurements, providing a clearer and more reliable estimation of the system's actual state. This feature is especially valuable in industrial and engineering applications, where precise control and accurate system state knowledge are crucial for optimal performance.

In the context of the RIG 38-100 Process, implementing a Kalman Filter allows for effective noise reduction in measurements such as fluid levels and flow rates, which are typically subject to disturbances. The Kalman Filter combines information from multiple sources, including sensor readings and system dynamics, to produce an optimal estimate of the state, ensuring that the control system can respond more accurately to changes in the process.

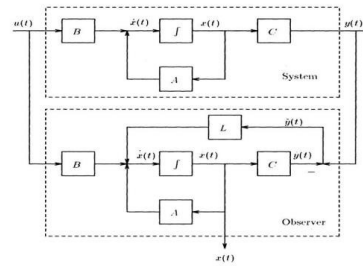


Figure 3. Feedback Estimator

2.6. Water Level System

The Water Level System Tank is a system where there is a storage tank which will later be filled with water through a pipe located above the storage tank, and there is also a drain pipe which is useful for disposing of the water below the storage tank.

Table 1. Parameter component plant

No	Parameter	Value
1	range of water discharge released by the pump	0-13,2 L/min
2	pump input range	0-12V
3	pump diameter	1 cm
4	length of pipe from pump to tank	292 cm
5	length of pipe from pump to Servo level	157,5 cm
6	Voltage range output level sensor	0-10V
7	range of water heights in the tank	0-70cm
8	Voltage range output flow sensor	0-5V
9	flow sensor water flow rate range	0-16L/min
10	tank volume (p.l.t)	18.13.14,5
11	water height at working point (ho)	7,5 cm
12	Water debit at working point (Qo)	13,7 cm/s

2.7. Linier Quadratic Gaussian (LQG)

This research uses a controller with the Linear Quadratic Gaussian (LQG) method. This controller is useful for maintaining the balance of the Level value according to the Set Point. Either when a load is applied or when there is no load. Ultimately, the Level value will be at the Set Point value.

The first step is to find the value of the weighting matrices Q and R. The Q and R matrices must be matrices with the correct dimensions. Determining the Q and R matrices uses a trial and error system, where you try the appropriate matrices until you can find Q and R values that are good and can actually reach the set point values that have been determined. At the trial and error stage, the appropriate Q and R values are obtained, namely:

$$Q = \begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 2000 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}; R = [0]$$

After determining the weighting matrix, the next step is to enter the matrix equation into the Ricatti equation algebra.

$$-PA - A^T P - Q + PB R^{-1} B^T P = 0$$

Calculations to find the matrix p can use one of the functions in Matlab

$$P = \text{are}(A, B * R^{-1} * B^T, Q)$$

Where A^T will be multiplied by the value of P that is sought, then added by the value of P multiplied by the value of A, then subtracted by the value of B multiplied by the value of R^{-1} multiplied by the value of B^T and also B then added by Q. And the P matrix is obtained as follows:

$$P = \begin{bmatrix} 0,0098 & 0,0115 & 0,0057 & 0,0010 \\ 0,0115 & 1,1769 & 0,5836 & 0,1012 \\ 0,0057 & 0,5836 & 0,5745 & 0,1012 \\ 0,0010 & 0,1012 & 0,1170 & 0,0578 \end{bmatrix}$$

After knowing the value of P, now look for the matrix value of K

$$K = R^{-1} B^T P$$

$$K = [98.1815 \quad 114.6428 \quad 57.4429 \quad 10.0379]$$

After knowing the value of P and K, now look for the matrix value of U

$$U = -Kx$$

$$U = [-7.3636 \quad -8.5982 \quad -4.3082 \quad -0.7528]$$

After knowing the value of P, K and U, now look for the value Kalman filter

$$L = P C^T R^{-1}$$

$$L = \begin{bmatrix} 0.3624 \\ 36.5320 \\ 42.2226 \\ 20.8617 \end{bmatrix}$$

After getting the variables needed to design the controller, the next step is to create a controller model in Simulink.

The linear plant simulation in Figure 4 is carried out by combining the state space matrix with the LQG controller. The x matrix is used as input for the plant. After going through the K matrix which is the LQG controller. Later the output signal will be filtered through a filter to determine whether noise occurs or not. If noise occurs it will be filtered and so on. By entering u as a comparator which produces a control signal. If a disturbance occurs in the plant or the control signal value is not as desired, K will repair it. This process repeats until it gets an output that matches the Set Point.

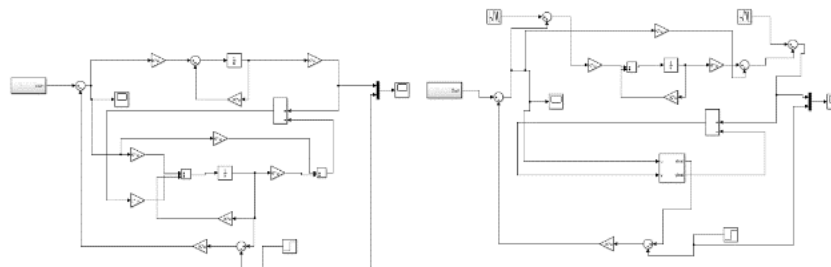


Figure 4. Simulation plant with LQG Controller

3. Results and discussion

3.1. Simulation Matlab

Testing was carried out with the help of Matlab software. Matlab requires a program that is used to run the simulation. The following is the program or code used to run the simulation:

% The ABCD value is obtained from changes in Transfer Function to State Space

```
A=[-3.0030 -2.3079 -0.3440 0.0378;1 0 0 0;0 1 0 0;0 0 1 0];
B=[1;0;0;0];
C=[0 0 0 0.0361];
D=[0];
% Desired Plant Output Value
X=[75];
%The Weighting Matrix
Q= [10000 0 0 0;0 2000 0 0;0 0 1000 0; 0 0 0 100];

R = [1];
% Noise Value
Qa = [0 0 0 0;0 25 0 0;0 0 75 0;0 0 0 100];
Ra = [0 0 0 0;0 25 0 0;0 0 75 0;0 0 0 100];
% Formulas to get the Ricatti equation
%[A^TP+PA-PBR^-1B^TP+Q=0]
RI=inv(R);
BT=transpose(B);
AT=transpose(A);
CT=transpose(C);
P = are(A,B*RI*BT,Q);
% Finds the K value with the selected Performance Index%[K=R^-1B^T]
K = RI*BT*P
% Define control signals
U = -K*X
% Determining Kalman Filter Values
L = P*CT*RI
```

3.2. Waveform

- Simulation without load

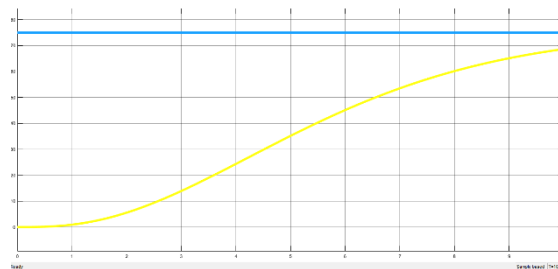


Figure 5. waveform without load

This graph depicts the response of a system tested in a no-load simulation. The horizontal blue line shows the condition of the system which reaches a constant value or steady state at a point around 90 on the vertical axis, which may reflect the reference or target value that the system wants to achieve in a stable state. The progressively increasing yellow line shows the system's response to time, which starts from zero and increases slowly in the absence of external disturbances or loads. This smooth increase indicates that the system is working in ideal conditions without any external factors influencing it.

In the context of a no-load simulation, this graph provides an illustration that the system is able to reach a stable value slowly and shows stable performance under these conditions. A less sharp or exponential response indicates that the system has smoother, more controllable characteristics, without the large fluctuations often seen under load conditions. This shows that the system works efficiently and can reach equilibrium in a reasonable time without any additional load that worsens its performance.

- Simulation with load

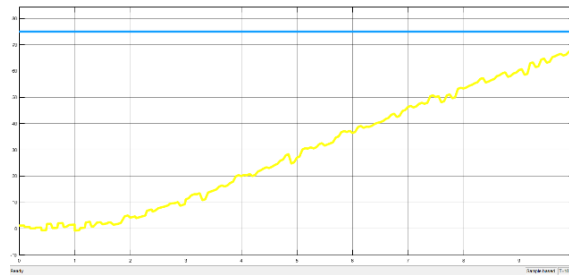


Figure 6. waveform with load

Based on the graph you just uploaded, there are several differences that can be seen when compared to the previous graph. The horizontal blue line still shows a constant value around 90 on the vertical axis, which reflects the system's stable or target value. However, the yellow line this time shows more irregular and jumping behavior, indicating that the system is experiencing fluctuations or disruptions, even though it remains improving overall.

The main difference seen is that in this second graph, the system does not show as subtle an improvement as before. Instead, there are small spikes that indicate instability or disturbances that affect the system's response. These fluctuations can be caused by external factors that influence system performance or more dynamic simulation conditions. Although the system continues to strive towards a stable value, these spikes show larger variations compared to the previous no-load simulation.

4. Conclusion

- **Determination of Gain (K):** The gain value (K) is determined through the appropriate selection of the Q and R matrices in the Linear Quadratic Regulator (LQR) design. By carefully tuning these matrices, an optimal feedback gain is achieved, resulting in the desired performance of the system. The choice of Q and R matrices plays a pivotal role in balancing the trade-off between state regulation and control effort, ensuring that the system behaves as expected under various operating conditions.
- **System Performance and Output Response:** The output response of the system has been successfully aligned with the initial objectives, particularly in stabilizing the water level at the predefined set point. Through the implementation of the LQR and Linear Quadratic Gaussian (LQG) control systems, the water level remains consistently stable at the target value of 75 units, demonstrating the effectiveness of the designed control strategy in maintaining process stability. This result highlights the efficacy of optimal control methods in real-world industrial applications, where precise and reliable control is essential.

Credit authorship contribution statement

Author Name: Conceptualization, Writing – review & editing. **Author Name:** Supervision, Writing – review & editing. **Author Name:** Conceptualization, Supervision, Writing – review & editing.

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