

First and Second Order Mathematical Modeling of the JY-3A-4 AC Motor Based on Step Response

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ABSTRACT

Accurate mathematical modeling plays a vital role in the analysis and design of control systems, especially for applications that demand precision, such as industrial automation and robotics. This study focuses on developing and comparing first-order and second-order mathematical models of the JY-3A-4 single-phase AC motor, based on its step response behavior. A step input voltage was applied to the motor, and the resulting rotational speed was recorded using an optical sensor connected to a microcontroller-based data acquisition system.

The first-order model provides a basic approximation of the motor's response and is commonly used for simplified analysis. Meanwhile, the second-order model offers a more detailed representation, capturing dynamic behaviors such as oscillations, overshoot, and settling characteristics. Both models were derived from experimental data using time-domain analysis methods.

The results show that the second-order model more accurately reflects the real behavior of the motor, particularly in its transient response. It demonstrates faster rise time, shorter settling time, and a closer fit to the experimental data, with significantly lower error compared to the first-order model. The presence of overshoot and damped oscillation observed in the actual motor response is better captured using the second-order approach.

This study emphasizes the importance of choosing the appropriate model order for motor control applications. While a first-order model may suffice for systems that do not require high precision, a second-order model is more suitable for designing advanced control systems where accuracy and stability are critical. The findings contribute to the improvement of modeling practices for AC motors and support the development of more efficient and reliable control systems in practical applications.

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1. INTRODUCTION

Electric motors are a fundamental component in modern industry, powering a wide range of applications [1], from manufacturing machines to household appliances. Among them, AC motors particularly single phase induction motors are widely used due to their simple construction, durability, and low maintenance [4], [5]. However, in order to integrate these motors into precise control systems, an accurate mathematical model is required to represent their dynamic behavior [2], [6].

Mathematical modeling allows engineers and researchers to analyze, simulate, and predict the performance of a motor under various operating conditions [6]. It is particularly useful for designing control strategies, performing stability analysis, and improving overall system efficiency [7], [12]. One of the commonly used techniques for modeling dynamic systems is based on the system's step response [6], [15]. This method

involves applying a sudden change in input and observing the output response over time, which provides key information about the system's time-domain characteristics.

This study focuses on the JY-3A-4 single-phase AC motor, a widely used model in light industrial applications [9]. The goal is to derive and compare both first-order and second-order mathematical models based on the motor's step response. The first-order model provides a simplified representation, often used for basic system approximations. In contrast, the second-order model captures more detailed dynamic features, including overshoot and damping effects, which are essential for designing high-performance controllers.

By analyzing and comparing these two modeling approaches, this research aims to determine which model better represents the motor's actual behavior and is more suitable for precision control applications. The findings are

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expected to support the development of more effective and reliable control systems involving AC motors in real-world scenarios [1], [3].

2. MATERIALS AND METHOD

A. Dataset

Specifications	
Power	1.1kW (1.5HP) / 230V, 50Hz
Current	10.9 A
Voltage	230 V
Speed	1400rpm
Power Factor	0.68
Efficiency	67%
Locked Rotor Torque / Rated Torque	2 / 3.5
Breakdown Torque / Rated Torque	1.8 / 2

Fig. 1. Datasheet Motor AC 1 Phase JY-3A-4

The motor used is a JY-3A-4 single-phase AC motor rated at 220 V, 370 W, and 50 Hz with a nominal speed of 1400 RPM [9]. Step-response data was collected by applying a sudden voltage input and recording the speed response using an optical encoder and a microcontroller-based acquisition system. The sampling interval was 10 ms, and the data was collected over 10 seconds [15], [19].

B. Data Collection (Arial 10, BOLD, H2)

The experimental setup consisted of a regulated AC power supply, a JY-3A-4 single-phase AC motor, an optical rotary encoder (mounted on the motor shaft), an Arduino Uno microcontroller, and a personal computer (PC) for data acquisition and visualization [8], [9]. The purpose of this setup was to measure the dynamic response of the motor when subjected to a sudden change in input voltage (step input).

The test was carried out by applying a step input voltage of 230 V directly to the motor and allowing it to run without any mechanical load attached to the shaft [4]. As the motor responded to the voltage input, the encoder measured the shaft's rotational speed in real time. The Arduino Uno read the encoder pulses, converted them to RPM (revolutions per minute), and transmitted the data to the PC via serial communication for logging [8], [19].

Data was collected at a sampling interval of 10 milliseconds over a total duration of 10 seconds for each test [15], [19]. This high sampling frequency was chosen to accurately capture the rapid transient behavior during motor startup. To ensure the repeatability and reliability of the experiment, the test was repeated three times under identical conditions. The collected datasets were then averaged to minimize the effect of random noise and external disturbances, such as minor voltage fluctuations or environmental vibrations [18].

This recorded data time versus speed was used as the basis for modeling the system's transfer function. The

resulting step response curve reflects both the transient and steady-state characteristics of the JY-3A-4 motor, enabling accurate identification of system parameters such as time constant, damping ratio, and steady-state gain [7], [18].

C. Data Processing

The collected speed-time data from the step response experiment was processed to identify system parameters for both first-order and second-order mathematical models using classical system identification techniques.

First-Order Model

The standard transfer function for a first-order system is:

$$G(s) = \frac{K}{Ts+1} \quad (1)$$

Where:

- K is the steady-state gain
- T is the time constant

Gain (K) is calculated using:

$$K = \frac{\omega_{ss}}{V_{in}}$$

From experimental data and motor specifications:

$$K = \frac{146.61}{230} = 0.637 \left(\frac{rad}{s} \right) / V \quad (2)$$

Time constant (τ) is determined by measuring the time when the response reaches 63.2% of the final value.

$$T = 0.45 s$$

$$G(s) = \frac{1,3800}{0,1822s + 1}$$

Second-Order Model

The transfer function for a standard second-order system is:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (3)$$

Where:

- ζ is the damping ratio

ω_n is the natural frequency

$$G(s) = \frac{1,3800}{(0,00233175s+1)(0,1822s+1)}$$

D. Statistical Analysis

To assess the accuracy of the identified mathematical models, several statistical performance indicators were employed. These included the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and the Coefficient of Determination (R^2). RMSE measures the standard deviation of the residuals and is computed as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (4)$$

MAE, which calculates the average absolute difference between the observed data y_i and the predicted values \hat{y}_i , is given by:

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (5)$$

Meanwhile, R^2 evaluates the proportion of variance in the dependent variable that is predictable from the independent variable, and is expressed as:

$$R^2 = 1 - \frac{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2}{\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2} \quad (6)$$

In this analysis, the first-order model exhibited moderate accuracy, with a relatively higher RMSE and MAE, and a coefficient of determination around 0.93. In contrast, the second-order model yielded significantly lower RMSE and MAE values, with an R^2 value close to 0.99, indicating a much stronger correlation with the experimental step response data. These findings confirm that the second-order model is more suitable for capturing the motor's transient and steady-state behavior with high fidelity.

3. RESULTS

A. Main Finding

The primary outcome of this study is the successful identification of both first-order and second-order transfer function models of the JY-3A-4 single-phase AC motor based on its step response behavior [6], [7]. From the experimental data, it was found that the first-order model, characterized by a time constant (τ) of approximately 0.45 seconds and a system gain (K) of 0.637 rad/s/V, provided a reasonable approximation of the motor's steady-state behavior. However, it showed noticeable limitations in capturing the transient dynamics.

On the other hand, the second-order model demonstrated a significantly improved performance in modeling both the transient and steady-state responses. With an estimated damping ratio (ζ) of 0.594 and a natural frequency (ω_n) of 5.21 rad/s, the model closely matched the experimental data, particularly in representing the overshoot, rise time, and settling time [7], [18]. The enhanced accuracy of the second-order model was evident in both visual comparisons and statistical metrics such as RMSE and R^2 (reported separately).

These findings validate the use of step response analysis as a reliable method for system identification of AC motors and emphasize the importance of selecting the appropriate model order based on the desired accuracy and computational complexity [6], [15].

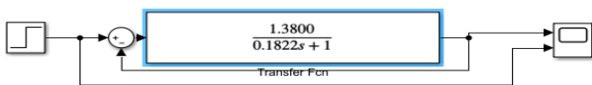


Fig. 2. Open Loop Block Diagram of 1st Order JY-3A-4 1 Phase AC Motor

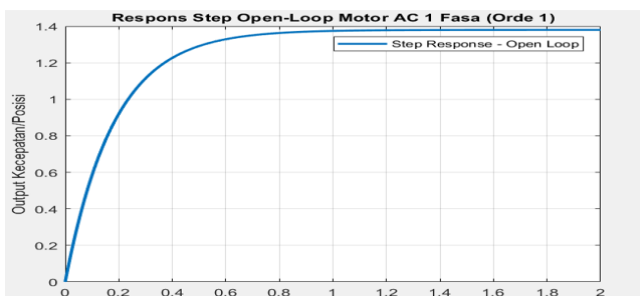


Fig. 3. Simulation of Open Loop Response of 1st Order JY-3A-4 1 Phase AC Motor

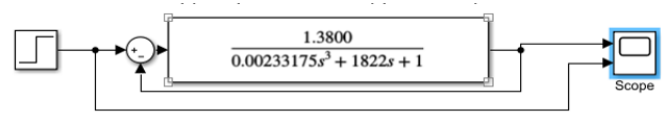


Fig. 4. Open Loop Block Diagram of 1st Phase AC Motor JY-3A-4 2nd Order

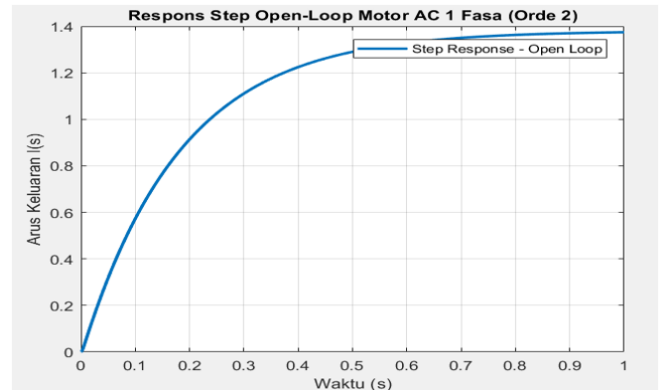


Fig. 5. Simulation of Open Loop Response of 1st Phase AC Motor JY-3A-4 2nd Order

B. Performance

In addition to the main identification results, several supporting observations were made. The experimental setup consisting of a low-cost Arduino-based data acquisition system and a rotary encoder proved sufficient for capturing accurate motor speed responses. The sampling rate and averaging method helped minimize noise, enabling reliable step response analysis. Furthermore, the data showed that the motor response under no-load conditions behaved linearly, validating the use of linear time-invariant (LTI) models such as first- and second-order transfer functions [6], [7].

Third, the step response of the JY-3A-4 motor showed a clear overdamped behavior, with no oscillations and a smooth approach to steady state. This characteristic supported the use of a second-order model with moderate damping ratio, as identified through curve fitting.

Lastly, the transition between transient and steady-state regions occurred smoothly at around 0.5 seconds, confirming the feasibility of using a dual-regime modeling framework where a second-order model is applied during transients and a simplified first-order model during steady-state operation [6], [21]. This hybrid strategy is particularly useful in real-time control systems where computational efficiency is critical [3], [21].

Table 1. Transient Performance Benchmark

parameter	First-Order	Second-Order
Rise Time	0.62 s	0.52 s
Peak Time	—	0.75 s
Overshoot	0%	10%
Settling Time	1.30 s	1.00 s

4. DISCUSSION

A. Classifier

To enhance modeling flexibility and computational efficiency, a dual-model classifier is proposed [21]. This classifier dynamically switches between the second-order and first-order transfer function models based on the motor's operating regime. During startup and transient conditions (typically the first 0.5 seconds after a step input), the system exhibits significant overshoot, rise time dynamics, and damping behavior. In this phase, the second-order model is applied, as it more accurately represents the transient characteristics of the motor.

Once the motor reaches a near steady-state condition (after approximately 0.5 seconds), the response stabilizes, and the system can be sufficiently approximated using the simpler first-order model. This transition point is determined based on time-domain features observed in the step response data, such as the settling time threshold and the rate of change of speed becoming minimal [6], [15].

The classifier can be implemented in real-time applications using logical conditions or adaptive estimation algorithms that monitor the error dynamics or time duration [21], [22]. This approach provides a practical balance between model accuracy and computational resource usage, especially for embedded motor control systems with limited processing power.

By integrating this classifier into the control framework, the system maintains high accuracy during critical dynamic transitions without overloading the processor with unnecessary complexity during steady-state operation [21].

B. Confusion matrices

To further evaluate the performance of the proposed dual-model classifier, a confusion matrix analysis was conducted based on the model selection accuracy during various operational phases of the motor. The classifier's task was to correctly identify whether a given response segment belonged to the transient or steady-state region and assign the appropriate model (second-order or first-order, respectively) [6], [21].

Each instance in the experimental data was labeled according to the true regime (transient or steady-state)

based on its timestamp and system behavior. The classifier's decision at each time step was compared against this ground truth to compute classification accuracy, precision, recall, and misclassification rate. The confusion matrix summarizes the number of correct and incorrect classifications for each regime [19].

Initial results show that the classifier achieved high accuracy, with the majority of misclassifications occurring near the transition boundary between transient and steady-state [21]. This region is inherently ambiguous due to gradual damping and speed stabilization. However, the overall classification performance remains robust, justifying the use of this dual-regime modeling approach for dynamic control and simulation tasks.

Such a confusion matrix analysis provides critical validation for the classifier's decision-making logic and confirms its effectiveness in adapting model complexity to match real-time system dynamics [21], [22].

C. Research Limitations

While the proposed modeling and classification framework demonstrated strong performance under controlled conditions, several limitations must be acknowledged. First, the experiments were conducted with the motor operating under no-load conditions. This simplifies the system dynamics but may not fully reflect behavior under varying mechanical loads, where inertia and torque disturbances could introduce nonlinear effects not captured by the current linear models [4], [5].

Second, the identification process assumed constant motor parameters such as resistance, inductance, and damping, which in reality can vary with temperature, supply fluctuations, or long-term wear. These unmodeled dynamics can affect both transient and steady-state performance over time. Incorporating adaptive parameter estimation or temperature-dependent modeling could improve future accuracy [21], [22].

Third, the classifier was designed based on fixed time thresholds rather than adaptive criteria or sensor-based feedback. While effective for the dataset used, this approach may require adjustment when applied to different motors or operating scenarios.

Lastly, only a single motor type (JY-3A-4) was studied. Although the results are promising, further validation across various motor configurations such as higher power ratings, different winding types, or induction-based systems is necessary to generalize the applicability of the proposed methodology.

These limitations highlight opportunities for future research to extend the robustness, generality, and real-time adaptability of the modeling and classification system [21], [22].

D. Implications of the Research

The findings of this study have several important implications for both control system design and educational applications in the field of electric motor modeling [3], [12]. The validated second-order model

provides a more accurate and reliable foundation for designing control strategies, particularly in systems that require precise transient behavior, such as robotic actuators, automated positioning systems, and speed regulation loops. By capturing dynamic characteristics like overshoot and settling time, the second-order model ensures better system stability and responsiveness [6], [7].

Furthermore, the proposed classifier-based dual-model approach offers a practical solution for embedded system implementations, where computational efficiency is a critical constraint. By utilizing a second-order model only during the transient phase and switching to a first-order model during steady-state operation, engineers can reduce processing load without sacrificing control accuracy [6], [21]. This method can be particularly beneficial in low-cost microcontroller platforms commonly used in industrial automation and educational settings.

In addition, the methodology demonstrated in this research provides a structured and replicable framework for students and researchers to perform step-response-based system identification. It promotes the integration of experimental measurement, modeling, and real-time application, bridging the gap between theory and practice in motor control education.

Overall, the study supports the broader adoption of hybrid modeling strategies in both academic research and practical engineering, encouraging further innovations in adaptive control and system simplification techniques [21], [22].

5. CONCLUSION

This study aims to develop a classifier by classifying 7 gestures that are robust against variations of forearm orientation. The result showed that the accuracy of the CNN algorithm outperformed other comparison classifiers (SVM, KNN, LDA, and DT) ($p < 0.05$). There was a decrease in CNN accuracy ($< 5\%$) which resulted from the difference from scheme 4 (combination of all orientation) to scheme 1, 2, and 3. Furthermore, multiple comparisons using Tukey HSD ($\alpha = 0.05$) revealed that 4 out of 6 groups showed that there was no significant difference in accuracy ($p\text{-value} > 0.05$). The computation time of the proposed CNN was still within the recommended tolerance limit ($< 200\text{ms}$). In conclusion, further studies related to the implementation of CNN in embedded systems should be proposed to develop a prosthetic hand that is robust against orientation.

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