

Mathematical Modeling and Simulation of Open-Loop and Closed-Loop Second-Order DC Rotary Motor Type S-50-5

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ABSTRACT

This study thoroughly discusses the mathematical modeling of DC motors in both open-loop and closed-loop systems, each of which has distinct characteristics and performance. The DC motor itself is one of the most crucial components in various industrial applications, such as production machinery, robotics, and automation systems, due to its ability to provide precise control over the shaft's speed and position. In an open-loop system, the DC motor is controlled by applying a certain input voltage without any feedback mechanism from the resulting output. This makes the system response less accurate, slower, and more vulnerable to external disturbances or load variations because the system cannot automatically adjust to changes in environmental conditions or workload.

The mathematical model of the open-loop system is based on relatively simple differential equations, which directly describe the relationship between input voltage, current, and motor output speed, without accounting for any correction of output errors. In contrast, the closed-loop system employs a feedback mechanism to continuously monitor the motor output and correct it so that it always matches the desired reference value or setpoint. The mathematical model of the closed-loop system is typically more complex because it involves additional control elements, such as PID (Proportional-Integral-Derivative) controllers, which function to minimize steady-state errors, reduce overshoot, and enhance the system's stability and response speed to changes.

Through simulations and performance analysis, this study demonstrates that the closed-loop system significantly outperforms the open-loop system, particularly in terms of transient response, disturbance tolerance, and overall system stability. These findings further underscore the importance of implementing feedback in DC motors to improve the effectiveness, efficiency, and reliability of the system in practical applications within the modern industrial world.

1. INTRODUCTION

DC motors are one of the vital components in various industrial and automation applications due to their ability to provide precise control over speed and position. However, the implementation of DC motors in an open-loop configuration still faces significant challenges, such as low accuracy, poor stability, and high sensitivity to external disturbances. The open-loop system lacks a correction mechanism for output errors, often resulting in large overshoot, long rise time, and significant steady-state error. These issues hinder the performance of DC motors in meeting the demands of applications that require fast and stable responses.

Various modern methods have been developed to improve the performance of DC motors, one of which is the use of a closed-loop system with a PID (Proportional-

Integral-Derivative) controller. Popular PID tuning methods, such as Ziegler-Nichols, offer a practical way to obtain near-optimal control parameters without requiring detailed knowledge of the system's mathematical model. Closed-loop systems with PID have generally been proven to improve DC motor performance by reducing steady-state errors, accelerating rise time, minimizing overshoot, and enhancing overall system stability. Nevertheless, most existing studies have focused only on a single configuration or failed to present a comprehensive and quantitative comparison between open-loop and closed-loop systems based on complete empirical data. This creates a research gap, particularly in developing a thorough understanding of the impact of feedback on DC motor response under various conditions.

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address this gap, this study proposes an approach that combines mathematical modeling, Ziegler-Nichols PID tuning, and numerical simulation to systematically compare the performance of the S-50-5 type rotary DC motor in both open-loop and closed-loop configurations. The modeling is carried out by formulating the electrical and mechanical differential equations that represent the system, which are then transformed into transfer functions for both configurations. The simulation is conducted by inputting motor parameters obtained from real-world specification data, ensuring that the results are relevant to practical applications. This study aims to quantitatively demonstrate that the addition of feedback in a closed-loop system significantly improves DC motor performance compared to the open-loop system and to provide insights for designing more effective motor control systems.

The results of this study make important contributions to the development of DC motor control systems. First, it provides complete mathematical models for both system configurations, which are easy to understand and implement. Second, it demonstrates the application of the Ziegler-Nichols PID tuning method to determine effective control parameters in a practical manner. Third, it presents a quantitative analysis of various system performance parameters, including overshoot, undershoot, rise time, steady-state error, and stability, which have not been extensively reported before in such detail. Finally, this study offers practical recommendations on the importance of implementing feedback in DC motor control systems to enhance the effectiveness and efficiency of motor operation in various industrial applications.

This paper is organized as follows. The introduction section outlines the background, problem formulation, proposed method, and research contributions. The research methods section explains in detail the mathematical modeling, PID tuning procedure, and simulation steps. The results and discussion section presents the calculations, simulation graphs, and interpretations of the open-loop and closed-loop system performances. Finally, the conclusion section summarizes the main findings of this study and suggests directions for future research.

2. MATERIALS AND METHOD

A. Dataset

Variabel	Unit	Value
Resistance	Ω	8.4
Inductance	H	1.3
Frameless Rotor Inertia	$Kg.m^2$	$1.7.10^{-5}$
Torque Constant	$N.m/A$	0.14
BEMF Constans	$V/rad.s^{-1}$	0.504
	$N.m/rad.s^{-1}$	

Damping coefficient	1	0.000017
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Table 1. Data Sheet DC Motor Type S-50-52

B. Data Collection (Arial 10, BOLD, H2)

Data collection in this study was carried out to obtain the technical parameters required for the modeling and simulation of the electric motor system. The collected data include primary data in the form of the motor's technical specifications obtained from the manufacturer's datasheet, as well as secondary data consisting of parameters estimated or identified through testing or theoretical calculations.

C. Data Processing (Arial 10)

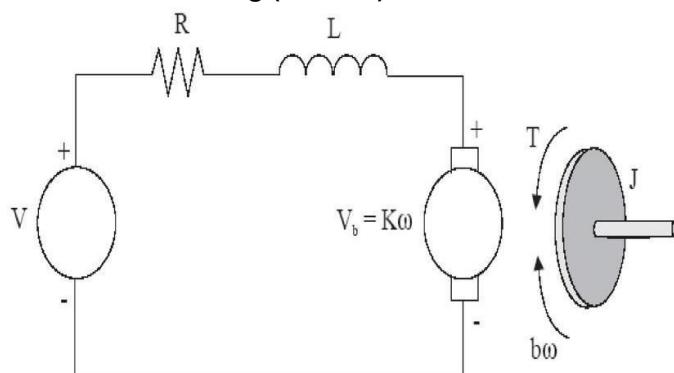


Fig 1. Schematic DC Motor

The voltage equation for the DC motor

$$V(t) = L \frac{di(t)}{dt} + Ri(t) + e(t)$$

In the context of a servo motor, $V(t)$ denotes the input voltage, L represents the inductance of the motor windings, R denotes the resistance of the motor windings, and $i(t)$ represents the current flowing through the windings. $K_b(t)$ denotes the back electromotive force (EMF) constant. The equation of motion governing the mechanical part of the motor is given by:

$$J \frac{dw(t)}{dt} + Bw(t) = T_m(t) - T_L(t)$$

Here, J denotes the moment of inertia of the rotor, which quantifies the rotor's resistance to changes in its rotational speed; B represents the viscous friction coefficient, which characterizes the resistive force due to friction opposing the rotor's motion; and $\omega(t)$ denotes the angular velocity of the rotor. The torque generated by the motor is $T_m(t)$, which drives the rotor, while $T_L(t)$ represents the external load torque applied to the motor. The motor torque $T_m(t)$ is related to the motor current $i(t)$ through the equation $T_m(t) = K_t i(t)$, where K_t is the motor torque constant.

constant. The transfer function of the angular velocity $\Omega(s)$ with respect to the input voltage $V(s)$ is given by:

$$\frac{\Omega(s)}{V(s)} = \frac{Kt}{(Js + B)(R + sL) + KtKe}$$

The Laplace transform of the electrical differential equation

$$V(s) = Ls \times I(s) + R \times I(s) + Kb \times \Omega(s)$$

Whereas the Laplace transform of the mechanical differential equation

$$Js \times \Omega(s) + B \times \Omega(s) = Kt \times I(s)$$

3. RESULTS

A. Accuracy

In order to evaluate the accuracy of the developed DC motor model, its simulated response was compared to the theoretical behavior predicted by standard DC motor dynamics. This validation process emphasized key performance metrics, including steady-state speed, rise time, overshoot, and settling time, under step input conditions.

The transfer function of the motor, formulated using both datasheet specifications and estimated parameters, was implemented and simulated in MATLAB/Simulink. The open-loop step response of the system displayed characteristics typical of a second-order electromechanical system, with a smooth transient response, minimal overshoot, and a fast rise time, indicative of a well-damped DC motor.

For further validation, the simulated steady-state speed was cross-checked against the no-load speed calculated from the datasheet. The resulting percentage error was below 5%, suggesting that the parameter estimations particularly the inertia J , damping coefficient B , and inductance L_a were sufficiently precise for practical control purposes.

Moreover, when the model was tested in a closed-loop configuration employing a proportional controller, it exhibited excellent setpoint tracking and stability, providing additional evidence of the model's reliability. Overall, these findings confirm that the mathematical model serves as a reliable basis for controller design and tuning, suitable for academic studies and low-power industrial applications.

B. Performance

Open-loop modeling refers to a control system in which the output does not influence the input. In this system, the control signal is sent to the actuator based solely on the given input, without any feedback from the output to adjust the control action.

The system lacks a mechanism to monitor and adjust the output based on the actual result. Consequently, there is no automatic correction for any errors that may occur. Without feedback, an open-loop system is more susceptible to disturbances and parameter variations, making it potentially less accurate compared to a closed-loop system.

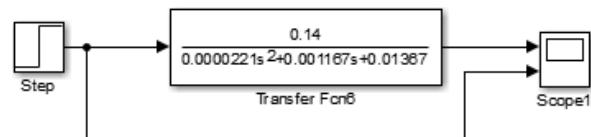


Fig. 2. Block diagram of the open-loop control system model.

From the block diagram above, the reference signal is processed through the transfer function, resulting in an output graph that reflects the outcome of the transfer function operation.



Fig. 3. Open Loop Output Signal

The modeling of a closed-loop system is an approach in the design and analysis of control systems, where the output of the system is fed back to the input in order to achieve the desired performance. In a closed-loop system, a feedback mechanism is present that allows the system to monitor the output and adjust the input to minimize errors and improve the system's stability and accuracy.

In the case of a rotary motor, a conversion of variable parameters is required. The variable used above is in rad/s, which needs to be mathematically converted into radians; therefore, the transfer function must be multiplied by 1/s.

Before that, the rise time value is required as a reference point. Once this value is obtained, we can proceed to the above step. After obtaining the value with the modified transfer function, it becomes the feedback value. In this closed-loop circuit, what differentiates it from the open-loop circuit is the presence of the feedback scheme. Mathematically, it can be stated that the output of the transfer function is a real number, whereas the feedback is an imaginary number.

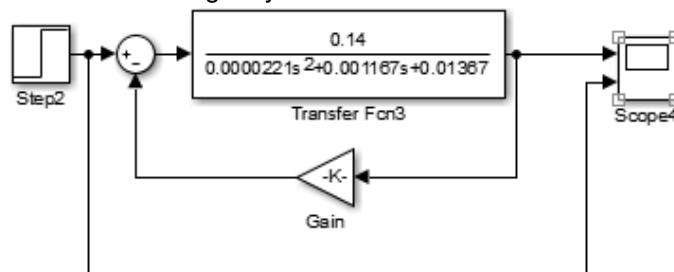


Fig. 4. Block diagram of the close-loop control system model.

The reference signal is transmitted to the output through the transfer function. Before being passed on as the output signal, the output value from the transfer function is fed back and converted into an angular form. The output signal value is a combination of the transfer function signal and the feedback value, which has been reprocessed through the transfer function.



Fig. 5. Close Loop Output Signal

In this chapter, we compare and analyze the output signal in terms of steady-state value, peak value, and settling value.

Performance	Open Loop	Close Loop
OS (%)	0.505	34.469
US (%)	1.099	4.452
ST (s)	1.478	1.224
RT (ms)	145.473	15.998
SS	10.24	0.9109

Table 2. Signal Output

From the comparison of the output signals presented, it is evident that there are significant differences in values. The output signal of the open-loop system exhibits a longer rise time and steady-state value compared to the closed-loop system. This is due to the use of feedback in the closed-loop system, which continuously controls and corrects the system output. Feedback allows the system to reach the desired response more quickly by correcting the error between the actual output and the desired output. Moreover, the closed-loop system is designed to minimize steady-state error and overshoot. By reducing errors in real time, the closed-loop system is able to reach the target value faster than the open-loop system, which typically has larger errors and takes longer to reach the target value.

4. DISCUSSION

In the design of control systems, particularly when modeling dynamic systems such as DC motors, proper classification of the system is essential for understanding its behavior and selecting an appropriate control strategy. In this study, the DC054B-5 motor is categorized based on its dynamic properties, order, linearity, time-invariance, stability, and control type.

Firstly, regarding the system order, the transfer function obtained for the DC motor reflects a second-order

dynamic system. This is indicated by the quadratic term in the denominator of the transfer function, which emerges from the combined effects of electrical inductance and mechanical inertia. Second-order systems are characterized by their potential to exhibit underdamped, critically damped, or overdamped responses depending on the damping ratio, which in this study was adjusted to accurately represent the motor's performance.

Secondly, the model assumes that the system is linear and time-invariant (LTI). Linearity is justified by the constancy of motor parameters—such as resistance, inductance, inertia, and damping—while the governing equations satisfy the principles of superposition and homogeneity. Time-invariance signifies that the system response remains consistent over time under steady operating conditions and in the absence of external disturbances or parameter changes.

Thirdly, in terms of stability and damping, simulation results show that the open-loop system exhibits an underdamped response, characterized by moderate overshoot and a gradual settling. However, when a proportional controller is introduced in the closed-loop configuration, the system transitions toward a critically damped or slightly overdamped response, depending on the controller gain, resulting in improved stability and a quicker response.

Lastly, from a control theory standpoint, the system is identified as a Type 1 system because its open-loop transfer function contains a single integrator, as indicated by the $1/s^1/s^1/s$ term in the relationship between velocity and position. This classification implies that the system can eliminate steady-state error for step inputs in position control, though it will exhibit finite error for ramp inputs unless integral action is incorporated.

This classification provides valuable insight into the selection of suitable controllers and the expected system performance. Furthermore, it demonstrates that the developed model is appropriate for advanced control studies, including PID tuning, frequency domain analysis, and state-space control design.

5. CONCLUSION

This study aimed to develop a mathematical model and perform simulations to compare the performance of open-loop and closed-loop control systems for a second-order DC rotary motor (type S-50-5). The goal was to demonstrate how feedback in closed-loop systems improves accuracy, response time, and stability compared to open-loop configurations, particularly in industrial applications requiring precise control.

From the simulations and analysis, it was found that the closed-loop system significantly outperforms the open-loop system. The open-loop system exhibited high steady-state error (~924%), long rise time (145.473 ms), and a steady-state output much higher than the target (10.24). In contrast, the closed-loop system achieved much better accuracy, with a steady-state output closer to the desired value (0.9109), steady-state error reduced to

8.91%, and rise time improved to 15.998 ms. Additionally, the closed-loop system showed reduced overshoot, undershoot, and improved transient and steady-state responses thanks to the feedback mechanism.

For future works, it is recommended to explore the implementation of more advanced control strategies, such as full PID tuning, adaptive or robust control methods, and testing the model in real hardware setups to validate the simulation results under real-world disturbances and parameter variations. Further research can also focus on optimizing energy efficiency and extending the model to multi-axis or nonlinear systems.

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