DC Motor Performance Optimization with Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) Methods

*Anisa Fitri Santosa
Bio-Industrial Mechatronics Engineering, National Chung Hsing University, taiwan
*Correspondence author: g112040521@mail.nchu.edu.tw

ABSTRACT

DC motors, also known as direct current motors, are electronic devices that are commonly used in a variety of contexts, both in industrial environments and in everyday life. To ensure optimal performance of DC motors, efficient control is required. To achieve this goal, signal optimization on DC motors is carried out through the application of the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) methods in the control system. This study aims to analyze and compare various technical responses that have been simulated through the application of control systems. LQR and LQT were chosen as methods because they are both able to reach the optimal point and reduce errors in the device, so that the performance of the device can be adjusted to the user's preferences and produce efficient output. The object of this research is a DC motor that has a data sheet available. The advantages of DC motors include having no losses in the reactive power generated, generating considerable torque, easy to control linearly, and the ability of the controller to reduce delay time, rise time, time to reach a steady state, as well as the magnitude of surges and faults in the system. By using the DC motor data sheet, a transfer function can be built that produces the order 1 and order 2 models as the basis for implementing the four control system applications. Data collection is carried out through direct research and observation to observe the results of experiments. The results of the study are explained through narratives, tables, and diagrams.

Keywords: *Linear Qudratic Regulator, Linear Quadratic Tracking, DC Motor*

I. INTRODUCTION

Due to the need for more efficient systems, the issue of optimal control has gained significant attention today. This is due to the fact that the selection of performance and engineering indices is necessary to produce a control system that operates within the limits of physical constraints. The purpose of the optimization of the control system is to find rules for system decision-making that will reduce the rate of errors caused by the system[1].

On this occasion, the author conducted a discussion focused on optimal control methods, namely Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT), which are often used in space-based systems[2][3]. The LQR control method involves determining two main parameters, namely the Q and R weight matrices, which must be carefully selected in order to produce optimal control actions according to the desired objectives[4]. Examples of applications of the LQR method include regulating the speed of induction motors, controlling frequencies in generator power plants, and controlling quadcopter drones. The integration of this LQR method is crucial in the discipline of system optimization, as it helps to reach the optimal point and reduce errors in equipment so that the performance of the equipment can be tailored to the user's preferences. LQT, on the other hand, is a linear control system that focuses on following a desired reference or path in a system. The Linear Quadratic Tracker (LQT) is a model-based tracking control mechanism that uses linear state feedback along with additional forward feed control elements[5][6]. This forward feed control element depends on the reference signal vector. Typically, LQT is used in the context of optimization issues related to path tracing or referencing in a system[7][8].

By utilizing these two methods, the author explains the use of the LQR method in DC motor systems, which is supported by data from datasheets[9]. The data from the datasheet will be integrated into the MATLAB script and simulated with MATLAB Simulink software to observe the step response. The DC motor used in this context is the Maxon DC Motor type RE40 Series 148877, which has information regarding the moment of inertia, motor constant, damping ratio, resistance, and inductance.

II. METHODOLOGY

2.1 LQR (Linear Qudratic Regulator)

LQR is an optimal control method that is often used in stateroom-based systems. The LQR controller involves two key parameters, namely the Q and R weight matrices, which must be carefully selected in order to produce optimal control actions according to the desired objectives[10][11]. Examples of the application of the LQR method involve regulating the speed of induction motors, controlling the frequency of power generation generators, and controlling quadcopter drones. By using the LQR method, the system will try to maintain the state around the predetermined reference point, thus maintaining the stability of the system even when there is interference or noise.

2.2 LQT (Linear Qudratic Tracking)

LQT's linear setting system follows (tracking) a predetermined path through input[12][13]. It will be explained how LQT can be used to regulate the output of the system so that it has minimal control energy and is close to the desired output. An observable linear system is depicted in Equation 1.

$$x(t)=A(t)x(t)+B(t)u(t) \quad (1)$$
$$y(t)=C(t)x(t)$$

With the error vector as in equation 2

$$e(t)=s(t)-y(t) \quad (2)$$

where e(t) is the error resulting from the difference between the desired input z(t) and the output of the system y(t), and the performance index, as shown in Equation 3 below.

$$J=\frac{1}{2}e(t_f)F(t_f)e(t_f)+\frac{1}{2}\int_{t_0}^{t_f} ¿¿ \quad (3)$$

With the matrix function P(t) must satisfy the equation

$$P(t)=-P(t)A(t)-A'(t)P(t)+A'(t)P(t)+P(t)B(t)R-1(t)B'(\quad (4)$$

Or it can be written:

$$0=-P(t)A(t)-A'(t)P(t)+A'(t)P(t)+P(t)B(t)R-1(t)B'(t)P \quad (5)$$

The Q and R matrices are assumed to match the expected performance levels of the system. After generating the Riccati equation, get a non-homogeneous vector differential equation by using Equation 6[14][5].

$$g(t)-[A-B(t)R-1B'(t)P(t)]'g(t)-C'(t)Q(t)s(t \quad (6)$$

After getting P(t) and g(t), only the K(t) gain still needs to be searched. K(t) can be obtained through Equation 7.

$$K(t)=R-1(t)B'(t)P(t) \quad (7)$$

Every parameter required for the LQT method has been obtained. Currently, Equation 8 should be used to find the optimal control u*(t).

$$u*(t)=-K(t)x*(t)+R-1(t)B'(t)g(t) \quad (8)$$

With x*(t) representing the feedback from the system. This feedback will then be multiplied by the K(t) control gain and added to the result of the multiplication R−1(t)B′ (t)g(t) to produce u*(t), which will later be used as system input to achieve the desired result.

2.3 Switch Function

The transfer function helps to see the characteristics of a system by comparing the input and output laplace functions with all the conditions that were initially considered zero[16][17]. The general form of the order 1 system is as follows:

$$G(s)=\frac{K}{\tau s+1}$$

As for the general form of the order 2 system, it can be expressed in the following standard form:

$$G(s)=\frac{\omega n^2}{s^2+2\varsigma\omega ns+\omega n^2}$$

2.4 Motor DC

A DC Electric Motor, otherwise known as a DC Motor, is a device that converts electrical energy into kinetic energy or movement. DC Motors are also often referred to as Direct Current Motors. As the name implies, DC motors have two terminals and require direct current voltage (DC - Direct Current) to move them[18]. These DC Electric Motors are commonly used in a variety of electronic and electrical devices that use DC power sources, such as mobile phone vibrators, DC fans, and DC power drills. The construction of the DC motor can be seen in Figure 2.5.

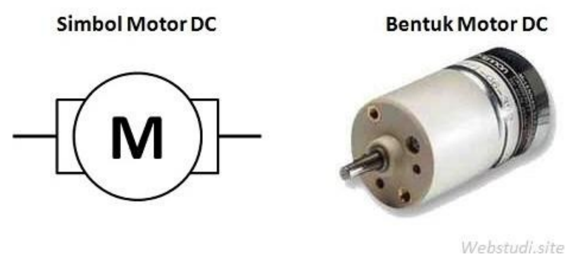


Figure 2. 1 DC Motor Construction

2.5 *Software* MATLAB

MATLAB is a programming platform that uses a matrix-based language, and is commonly used for data analysis, algorithm development, and modeling and application creation. The display of the MATLAB software is similar to that shown in Figure 2.3[19][20].

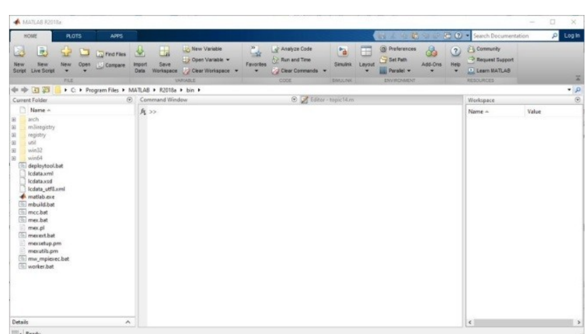


Figure 2. 2MATLAB view

In order to observe the response generated by DC motors, the authors utilize the Simulink feature in MATLAB. Simulink is a component in MATLAB that functions as a graphics-based programming tool.

Simulink has a major role in the creation of dynamic system simulations. This simulation is carried out using a functional diagram consisting of various interconnected blocks with their respective functions. Simulink serves as a modeling, simulation, and analysis tool for dynamic systems by utilizing a graphic-based user interface. Simulink consists of a variety of tools that can be used to analyze both linear and non-linear systems.

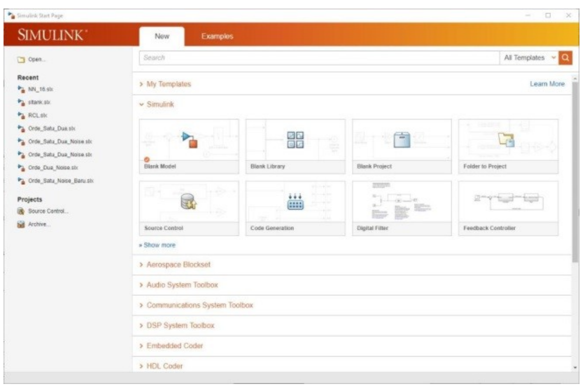


Figure 2. 8 MATLAB Simulink Display

2.5 Tools and Materials

The tools and materials used in this practicum are:

1. Laptop
2. Software Matlab
3. Maxon DC Motor RE40 series 148877 Datasheet

2.6 Calculation Analysis

Motor Specifications

- Motor Name = Maxon DC Motor RE40 series 148877
- τ = 184 mNm/A = 0.184 N/m
- Moment of Inertia (J) = 1380 kg.m2/s2
- Mechanical System Damping (B) = 4.39 ms
- Motor Constant (K) = 0.603 Nm/A
- Resistance (R) = 1.16 Ohm
- Inductance (L) = 0.000329 H

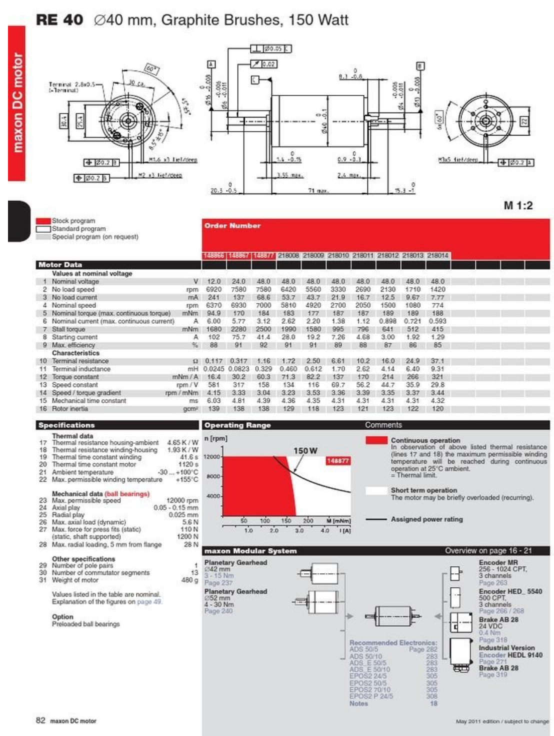


Figure 3.1 Maxon DC Motor RE40 series 148877 Datasheet

In addition, a mathematical model of the first-order system for Maxon DC Motor RE40 series 148877 can be obtained from the datasheet. A first-order system is a system that only changes once [10]. The modeling of the first-order system is described here.

General forms of order 1 transfer functions

$$G(s)=\frac{K}{\tau s+1}$$

Order 1 DC Motor

Based on the DC motor data sheet, the 1st order equation is obtained:

Where $\tau = K, i\ sehingga$

$$K=\frac{\tau}{i}=\frac{0{,}184}{3{,}12}\ v=0{,}0589$$

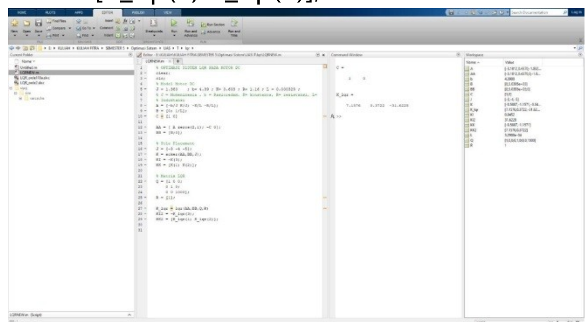
Equation of order 1 dc motor:

$$G(s)=\frac{0{,}0589}{0{,}184\,s+1}$$

2.7 Program Script Matlab

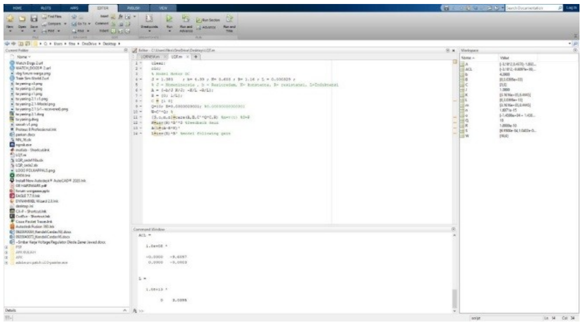
2.7.1 LQR

```
% OPTIMIZATION OF LQR SYSTEM ON DC MOTORS
Clear;
CLC;
% DC Motor Models
J = 1.380 ; b= 4.39 ; K= 0.603 ; R= 1.16 ; L = 0.000329 ;
% J = Moment, b = Ratio, K= constant, R= resistance, L=Inductance
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0]

AA = [ A zeros(2,1); -C 0];
BB = [B; 0];

% Pole Placement
J = [-3 -4 -5];
K = acker(AA,BB,J);
KI = -K(3);
KK = [K(1) K(2)];
% Matrix LQR
Q = [1 0 0;
    0 1 0;
    0 0 1000];
R = [1] ;

K_lqr = lqr(AA,BB,Q,R)
KI2 = -K_lqr(3);
KK2 = [K_lqr(1) K_lqr(2)];
```



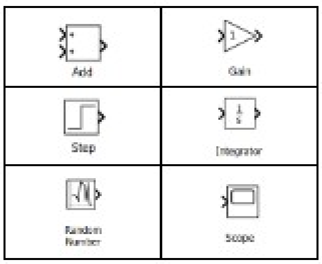
2.7.2 LQT

```
Clear;
CLC;
% DC Motor Models
J = 1.380 ; b= 4.39 ; K= 0.603 ; R= 1.16 ; L = 0.000329 ;
% J = Moment, b = Ratio, K= constant, R= resistance, L=Inductance
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0]
Q=10; R=0.0000000001; %0.00000000000001
W=C'*Q; %
[S,o,m,n]=care(A,B,C'*Q*C,R)
%m=v(t) %S=P
K=inv(R)*B'*S %feedback Gain
ACL=(A-B*K)'
L=inv(R)*B' %model following gain
```

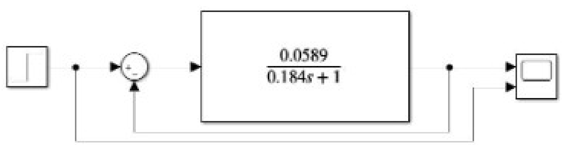


2.8 Maxon DC Motor RE40 series design 148877 on Simulink

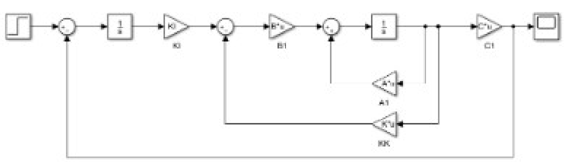
2.8.1 Components list



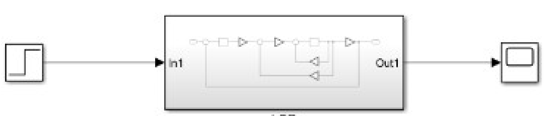
2.8.2 Maxon DC Series RE40 Series 148877 Order 1



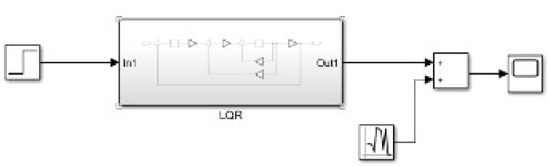
2.8.3 LQR Network



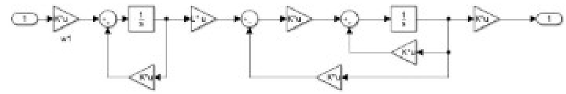
2.8.4 Noiseless LQR Subsystem Circuits



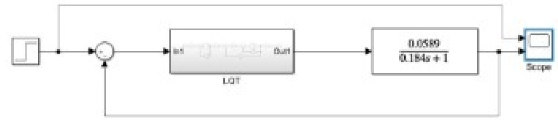
2.8.5 LQR Subsystem Circuits with Noise



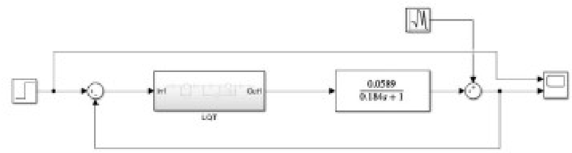
2.8.6 LQT Network



2.8.7 Noiseless LQT Subsystem Circuits



2.8.8 LQT Subsytem Network with Noise



III. RESULTS & DISCUSSION

3.1 Simulation Results of Maxon DC Motor RE40 series 148877 Order 1

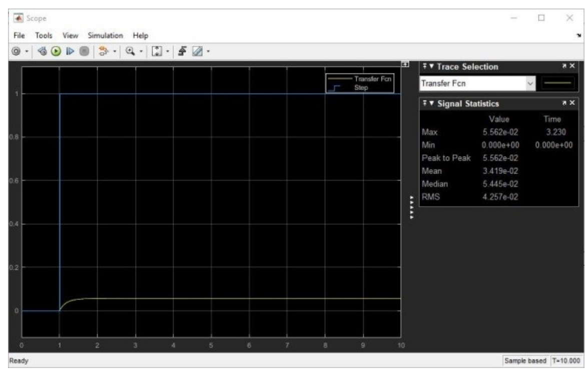


Figure 3. 1 Step Response Display

In Figure 3.1, there is an illustration of the step response of the Maxon DC Motor 148877 series without interference in the order one system (SISO). The simulation results show a stable step response graph with an amplitude of 5.507 (which did not reach the set point) and a rise time of about 377ms.

3.2 LQR Simulation Results without Noise

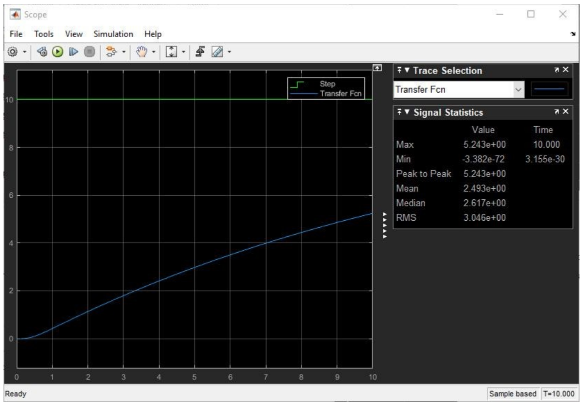


Figure 3.2 LQR Step Response Display without Noise

Figure 3.2 shows the step response of the Maxon RE40 series DC motor LQR without 148877 noise. The step response output of the BN-42 DC motor LQR reached an amplitude of 5 because it had not reached the setpoint.

3.3 LQR simulation results with noise

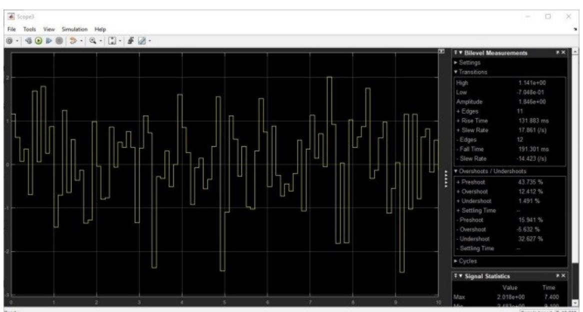
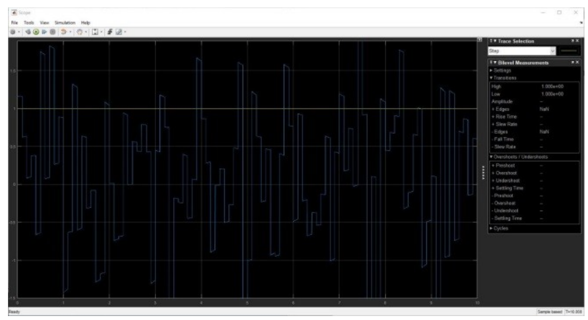


Figure 3. 3 LQR Step Response Display with Noise

Figure 3.3 shows the step response of the Maxon DC motor 148877 series motor which is controlled using the LQR method with noise. It can be seen that the step response of the LQR Maxon RE40 DC motor 148877 series has a fluctuating graph due to the influence of noise. The system reaches an amplitude of about 1.84, which has not yet reached the setpoint. In addition, the rise time reached a maximum value of around 191.301ms, with an overshoot of about 12.412%, and an undershoot of about 1.491%.

3.4 LQT simulation results

No noise



With noise

Figure 3. 4 LQT Simulation Results with Noise and No Noise

From the display on the scope, it can be concluded that the output of the DC motor system of the order 1 with the use of LQR & LQT methods and the presence of noise interference has succeeded in reaching the desired set point. However, the graph shows additional fluctuations caused by the input noise, so it does not run smoothly. The yellow graph line reflects the input value of the step input, while the blue graph line reflects the output of a 1st order DC motor system using the LQR & LQT method in the presence of noise interference.

IV. CONCLUSION

1. A DC motor datasheet equipped with moment of inertia values, motor constants, damping ratios, resistance, and inductance is required to generate a mathematical model of a 1st order DC motor and the variables required for LQR. After performing 1st order mathematical modeling calculations, it was found that the transfer function. The values of variables A, B, C, and K_lqr, which appear in the workspace, are found by running the matlab LQR script.
$$G(s)=\frac{0,05890}{184s+1}$$
2. The Step response results of Maxon DC motors 148877 series motors with order 1 show a stable step response graph with an amplitude of about 5.507, which indicates that it has not reached a setpoint of 1. The rise time was about 377ms, and the system experienced an overshoot of around 0.485% and an undershoot of around 1.226%. Meanwhile, the output of the step response of the Maxon DC Motor 148877 series reached an amplitude of about 4.037, which also did not reach the setpoint.
3. The results of the step response of the two systems can be compared and it is concluded that the Maxon DC Motor 148877 series motor system using the LQR method has not achieved optimal results compared to the Maxon DC Motor system series 148877 order 1. This can be seen from the fact that with the use of the LQR method, the step response of the Maxon DC Motor 148877 series has not reached a setpoint, has a graph that still tends to rise, and has an overshoot and undershoot with a small value.
4. In conducting simulations using Simulink in MATLAB software, it is necessary to use transfer functions derived from 1st order DC motor modeling. This transfer function is used to analyze the step response of the LQR & LQT system of DC motors. In order to create a mathematical model of a DC motor of the 1st order, the data present in the corresponding DC motor datasheet must be available. In this experiment, datasheets from Maxon DC Motor RE40 series 148877 used. The simulation results of the LQR & LQT system, utilizing a transfer function based on the mathematical modeling of Maxon DC Motors RE40 series 148877 order 1, show that the system is capable of achieving the output according to the desired set point.

V. CLOSING

1. Awards

research, especially those that fund your research. Include individuals who have assisted you in your studies: Advisors, Financial Supporters, or perhaps other supporters such as Proofreaders, Typists, and Suppliers who may have provided the material.

Researchers realize that without the support of various parties, the preparation of this community service journal will never be realized. So on this occasion, the researcher would like to express his gratitude to various parties who have participated. (This point can be readjusted by adding words or including the party you want to appreciate)

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