Linear Quadratic Regulator (LQR) and Linear Quadratic Tracker (LQT) System for DC Motor M66CE-12

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ABSTRACT

The purpose of system optimization is to determine the appropriate control signal so that the system output can maintain or follow the given reference, while minimizing or maximizing a certain performance index. Optimal control is divided into two main cases, namely the regulator case (known as Linear Quadratic Regulator or LQR) and the tracking case (known as Linear Quadratic Tracking or LQT). In this report, the author discusses the LQR and LQT methods as contained in the Automation Engineering course at PPNS entitled "System Optimization". In this study, the author uses Simulink to simulate the system created using the LQR and LQT methods. To obtain a mathematical model of a 1st order DC motor and the variables needed in LQR, a DC motor datasheet is required involving the value of the moment of inertia, motor constant, damping ratio, resistance, and inductance. By performing a 1st order mathematical modeling calculation, it can be compared and concluded from the results of the step response of the two systems. The results show that the M66CE-12 DC motor system using LQR has a more optimal performance compared to the M66CE-12 motor of order 1. With LQR, the step response of the M66CE-12 DC motor can reach the set point, show a stable graph, have a fast rise time, and show overshoot and undershoot with small values. Meanwhile, the step response produced by the LQT system also shows a more optimal performance compared to the two systems.

Keywords : *System, Motor, LQT, LQR*

I.INTRODUCTION

Current technological advances are developing very rapidly and have a significant impact on various aspects of life[1][2]. It is undeniable that the need for technology has increased rapidly in recent decades. As a result, there is fierce global competition, especially in industry. Many industry players build or improve their systems to produce superior products and compete with their competitors[3][4]. In this context, the problem of optimal control becomes very important, especially because of the increasing need for systems that have high performance. The concept of control system optimization involves the selection of performance indices and engineering that can produce an optimal control system, but still comply with existing physical constraints. In dealing with optimal control systems, the goal is to find a decision-making rule for a control system that can minimize the size of the deviation from the desired or ideal behavior[5][6].

System Optimization is a concept in modern control theory[7]. The goal of system optimization is to find the right control signal so that the system output can maintain or follow a given reference, while minimizing or maximizing a certain performance index. From the perspective of its objectives, optimal control is divided into two main cases, namely the regulator case (known as Linear Quadratic Regulator) and the tracking case (known as Linear Quadratic Tracking)[8][9]. In the tracking case, the system is designed to maintain the output to follow the given reference. This Linear Quadratic Controller requires linear properties in the system and the desired performance index is formulated in quadratic form.

In this report, the author discusses the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) methods which are part of the "System Optimization" course in PPNS. In this study, the author uses Simulink to simulate the system created by applying the LQR and LQT methods [19]. Simulink is a graphical extension of MATLAB that is used to model and simulate various systems. In Simulink, the system is represented in the form of a block diagram, including transfer functions, summing junctions, and virtual input and output devices such as function generators and oscilloscopes [20].

II.METHODOLOGY

2.1 MATLAB

MATLAB is a matrix-based programming language platform that is commonly used to create algorithms, analyze data, and create models and applications. Figure 2.1 shows the MATLAB program display[10].

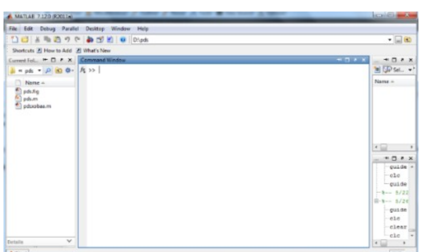


Figure 2.1 MATLAB display

The author uses the Simulink feature in MATLAB, one of its components that functions as graphical programming, in simulations to see the response produced by the DC motor[11][12].

Simulink has a primary function as a tool for creating dynamic system simulations. The simulation process is carried out through functional diagrams, which include connected blocks with their respective functions equally. Simulink functions as a tool for modeling, simulating, and analyzing dynamic systems by providing a graphical user interface[13]. This tool consists of various toolboxes that can be used to analyze systems that are both linear and non-linear[14][18].

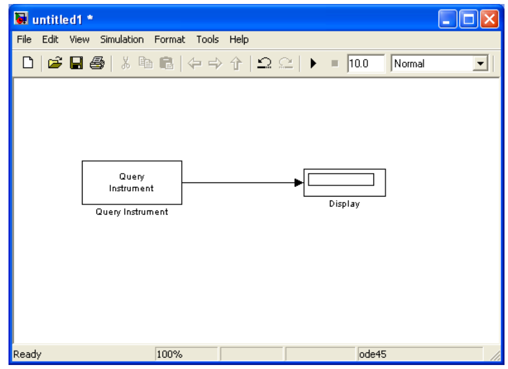


Figure 2.2 MATLAB Simulink View

2.2DC Motor Identification

Table 1. DC motor modeling

Specification	Units	M66CE-12
Maximum Voltage	VDC	12
Maximum Continuous Output Power	Watts	15
Maximum No-load Speed	Rpm	2700
Typical speed torque	Rpm	1800
Rated torque	Ncm	8
Peak torque	Ncm	25
Typical no load current	mA	120
Inertia Rotor	Kgcm2	0.214
Mechanical time constant	Msec	24.5
Torque constant	Nm/A	0.041
Constant Voltage	v/1000rpm	4.27
Rotor resistance	Ohm	1.9
Rotor inductance	Mh	1.0

Specification

- Motor Name = DC Motor M66CE-12
- t = 8 ncm = 0.08 N/m
- Rated Current = 2 A // current
- Voltage = 12 V
- Speed = 2700 rpm or 282.74 m/s

1st Order Modeling
The general form of a first-order transfer function:

$$G(s)=\frac{K}{\tau s+1}$$

- First Order DC Motor Modeling

Based on the DC motor datasheet, the first order equation is obtained [15]:
Where = K. i so that

$$K=\frac{\tau}{i}=\frac{0.08}{2}=0{,}04 \quad (1)$$

C(s) = System output.
R(s) = System input.
K = Overall Gain.
τs+1 = Time required to reach 63.2% (seconds) at first order. First order equation of DC motor:

$$G(s)=\frac{0{,}04}{0{,}08s+1} \quad (2)$$

- Modeling of 2nd Order DC Motor

The general form of a second-order transfer function:

$$G(s)=\frac{\omega n^2}{s^2+2\varsigma\omega ns+\omega n^2}$$

$$G(s)=\frac{2\pi f^2}{s^2+2\varsigma(2\pi f)s+2\pi f^2} \quad (3)$$

$$G(s)=\frac{2\pi 50^2}{s^2+0.214(2\pi 50)+2\pi 50^2}$$

$$G(s)=\frac{98596}{s^2+134{,}392s+98596}$$

2.3 Matlab Script Program for LQR System on DC Motor

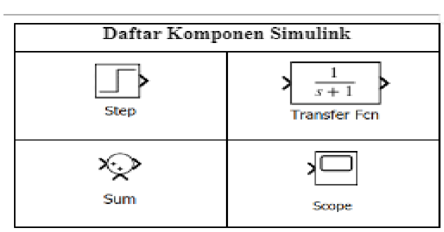
```
% LQR SYSTEM OPTIMIZATION ON DC MOTOR
clear;
clc;
% DC Motor Models
J = 49,430 ; b= 0.1 ; K= 0.144 ; R= 0.041 ; L = ;
% J = Momentum, b = Damping ratio, K = constant, R = resistance, L = Inductance
A = [-b/JK/J; -K/L -R/L];
B = [0;1/L];
C = [1 0]

AA = [ A zeros(2,1); -C 0];
BB = [B;0];
% Pole Placement
J = [-3 -4 -5];
K = acker(AA,BB,J);
KI = -K(3);
KK = [K(1) K(2)];
% LQR Matrix
Q = [1 0 0;
   0 1 0;
   0 0 1000];
R = [1] ;

K_lqr = lqr(AA,BB,Q,R)
KI2 = -K_lqr(3);
KK2 = [K_lqr(1) K_lqr(2)];
```

2.4 LQT System Matlab Script Program for DC Motors

```
% LQR SYSTEM OPTIMIZATION ON DC MOTOR
clear;
clc;
% DC Motor Models
J = 0.214 ; b= 0.1 ; K= 0.144 ; R= 0.041 ; L = 0.001;
% J = Momentum, b = Damping ratio, K = constant, R = resistance, L = Inductance
A = [-b/JK/J; -K/L -R/L];
B = [0;1/L];
C = [1 0]
Q=10; R=0.0000000001; %0.00000000000001
W=C'*Q; %
[S,o,m,n]=care(A,B,C'*Q*C,R) %m=v(t) %S=P
K=inv(R)*B'*S %feedback Gain
ACL=(AB*K)'
L=inv(R)*B' %model following gain
```

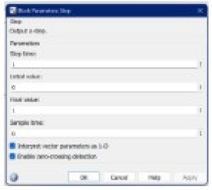
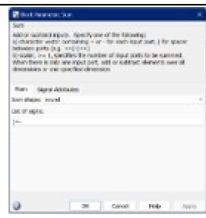
2.5 Designing a System in Simulink

Table 2 List of Required Components[16][17]

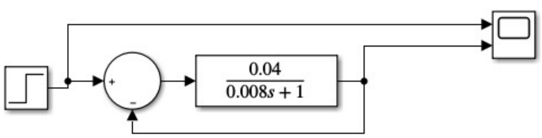


- Set the configuration of each component

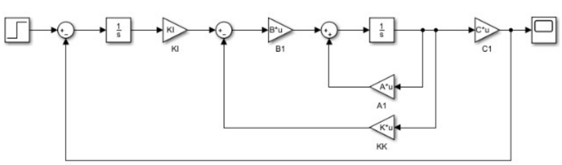
Table 3 List of Required Components

Konfigurasi Komponen		
STEP		
SUM		
TRANSFER FCN	 Orde 1	
	 Orde 2	

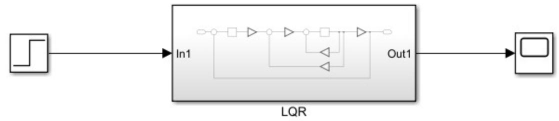
2.6 M66CE-12 DC Motor Circuit



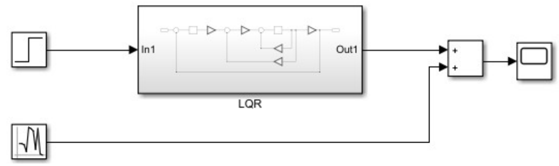
2.5 LQR circuit



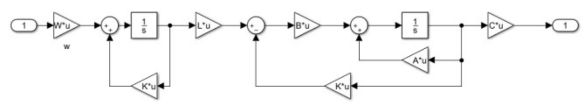
2.6 LQR Subsystem Circuit without Noise



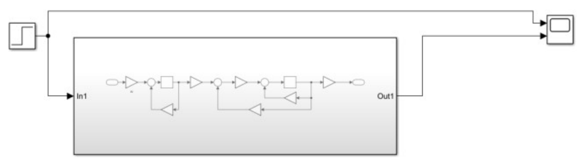
2.7 LQR Subsystem Circuit with Noise



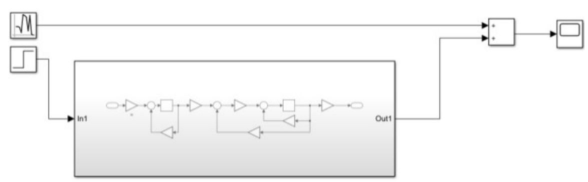
2.8 LQT circuit



2.9 Noise-free LQT Subsystem Circuit



2.10 LQT Subsystem Circuit with Noise



III.RESULTS & DISCUSSION

3.1 Matlab LQR Run Script Results

Command Window

```
C =

     1     0

K_lqr =

    8.9644    0.9658  -31.6228
```

3.2 Matlab LQT Run Script Results

```
Command Window

c =

     1     0

s =

   1.0e-03 *

    0.9694    0.0000
    0.0000    0.0000

o =

   1.0e+04 *

  -1.0315 + 1.0315i
  -1.0315 - 1.0315i

m =

   1.0e+05 *

    3.1621    0.0002
n =

   5.4136e-15

K =

   1.0e+05 *

    3.1621    0.0002

ACL =

   1.0e+08 *

   -0.0000   -3.1621
    0.0000   -0.0002

L =

   1.0e+13 *

          0    1.0000
```

3.3 Results of Simulation of Modeling of M66CE-12 Motor Order 1

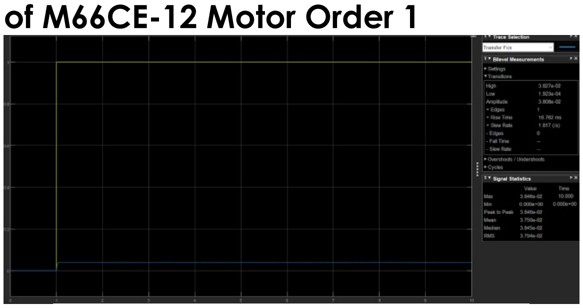


Figure 3.3 LQR Step Response Display

In Figure 3.3, there is a blue graph that has not reached the set point. The amplitude generated from this simulation is 3.808e-02. The recorded Rise Time is 16.762 ms, with an Overshoot of 0.495% and an Undershoot of 0.847%

3.4 Simulation Results of M66CE-12 Motor Modeling without LQR noise

Figure 3.4 Simulink results of M66CE-12 LQR DC motor without noise

Figure 3.4 shows the simulation results of the M66CE-12 DC Motor using the LQR method without noise. The yellow graph reaches the setpoint with a simulation amplitude of 9.900e-01. The resulting Rise Time is 1.100 s, with an Overshoot of 1.518% and an Undershoot of 0.505%.

3.5 Simulation Results of Modeling the M66CE-12 Motor Order 1 with LQR noise

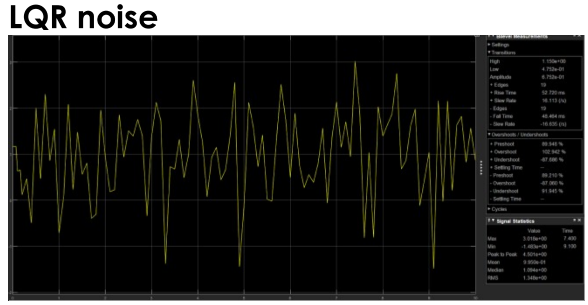


Figure 3.5 Display of 1st Order Step Response with noise

Figure 3.5 shows the step response display of the M66CE-12 DC motor with noise. The step response output of the LQR of the M66CE-12 DC motor shows a fluctuating graph due to the influence of the noise given. The system reaches an amplitude of 6.752e-01, so it has not reached the setpoint. The recorded rise time is 52.720 ms, with an overshoot of 102.942% and an undershoot of -87.686%.

3.6 Simulation Results of M66CE-12 Motor Modeling without LQT Noise

Figure 3.6 Display of 2nd Order Step Response with noise

From the data in Figure 3.6, the graph successfully reaches the setpoint. The system reaches an amplitude of 9.981e-01, indicating that the system has not fully reached the setpoint. Rise time reaches a maximum of 198.255 us, with an overshoot of 5.851% and an undershoot of 0.634%.

3.7 Simulation Results of M66CE-12 Motor Modeling with LQ Noise

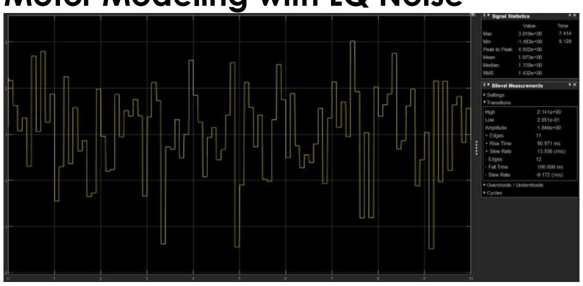


Figure 3.7 First Order Step Response Display

From the results in Figure 3.5, the step response display of the M66CE-12 DC motor with noise can be seen. The step response graph of the LQR of the M66CE-12 DC motor shows fluctuations caused by the introduced noise. The system reaches an amplitude of 1.846e-00, indicating that the system has not fully reached the setpoint. The rise time reaches a maximum value of around 90.971 ms, with an overshoot of 12.405% and an undershoot of 102.946%.

IV.CONCLUSION

To obtain the mathematical model of the 1st order DC motor and the parameters required for the Linear Quadratic Regulator (LQR), DC motor data needs to be available, including the value of the moment of inertia, motor constant, damping ratio, resistance, and inductance. By performing the calculation of the 1st order mathematical modeling, the transfer function G(s) = 0.04/(0.08 + 1) can be found. The results of the Matlab script execution for LQR and LQT provide variable values such as A, B, C, K_lqr, and so on, which appear in the workspace. From the analysis of the step response of the two systems, it can be concluded that the M66CE-12 DC motor system with the application of LQR produces more optimal performance compared to the 1st order M66CE-12 motor. The application of LQR allows the step response of the M66CE-12 DC motor to reach the setpoint, show a stable graph, have a fast rise time, and overshoot and undershoot with low values. However, the step response produced by the LQT system is more optimal compared to the two systems.

V.CLOSING

1.Awards

The researcher realizes that without the support of various parties, the preparation of the journal Improving DC Motor Performance Using the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) Approach will never be realized. So on this occasion the researcher would like to express his gratitude to the various parties who have helped, especially Mr. Anggara Nugraha Trisna for giving the researcher the opportunity to work on this journal, thank you also to Kak M. Fikri for helping to provide suggestions and explanations, thank you also to the family and friends who have supported the researcher in working on this journal, thank you also to the Spotify playlist

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