

Comparing Linear Quadratic Regulator (LQR) with Proportional-Integral-Derivative (PID) Controllers for Increasing Stability in DC Motor Systems

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ABSTRACT

A DC motor is a versatile type of motor widely applied in industries, robotics, and household appliances due to its broad speed regulation range and ease of integration. Among the various types of DC motors, the series DC motor stands out for its high starting torque. However, this characteristic also leads to significant challenges, including overshooting during initial start-up and instability under varying load conditions. For instance, at high torque, the motor's speed tends to decrease, while at low torque or no-load conditions, it often produces excessively high speeds. To address these issues and achieve accurate speed regulation with stable final results, a robust control strategy is required. Controllers play a pivotal role in minimizing overshoot and ensuring stability in motor performance. This research investigates the performance of two control methodologies—Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR)—through MATLAB-based simulations for regulating the speed of a series DC motor. In this study, motor speed is analyzed to evaluate the effectiveness of the controllers. The simulation results reveal that both PID and LQR controllers achieve minimal error rates. However, there are notable differences in their dynamic response. The PID controller demonstrates a faster rotor speed response time compared to the LQR controller. Nonetheless, the PID controller exhibits a significant overshoot of approximately 20%, whereas the LQR controller effectively eliminates overshoot altogether. This study contributes to the growing body of knowledge in control systems engineering, particularly in evaluating advanced controllers for industrial applications.

Key Word: series dc motor, LQR, PID, speed control, matlab

I. INTRODUCTION

DC motors are widely recognized for their versatility and broad speed regulation range, making them a popular choice in various applications such as industrial systems, robotics, and household devices [1]. Among the many control strategies developed to regulate DC motors, the Proportional-Integral-Derivative (PID) controller is one of the most extensively used due to its straightforward structure and ease of parameter tuning [2]. However, despite its widespread application, the PID controller has certain limitations, particularly in handling systems with high dynamic variations or minimizing overshoot [3][4].

In addition to PID, another advanced control strategy that has gained attention is the Linear Quadratic Regulator (LQR) [5][6]. The LQR controller, rooted in optimal control theory, offers the potential for improved system stability and performance under varying operational conditions. Recognizing these factors, this study aims to conduct a comparative analysis of the performance of PID and LQR controllers in regulating the speed of a DC motor [7][8][9].

Through this comparison, the research seeks to identify the most optimal control strategy for stability enhancement and dynamic response. By evaluating system responses using both controllers, the findings are expected to provide valuable insights into

the effectiveness of these control methods, contributing to the ongoing development of advanced control systems in the field of engineering.

II.METHODOLOGY

1. Parameter Motor DC

Parameter	Symbol	Value Unit
Moment inertia	J_n	0.0007046 Kg.m ²
Friction coefficient	B_n	0.0004 N.m/(rad/s)
Torque constant	K_i	0.1236 N.m/A
Reverse voltage constant	K_b	0.1236 V/(rad/s)
Total coil resistance	R_t	7.2 ohm
Total coil inductance	L_t	0.0917 H

2. PID

To effectively design and evaluate control systems for DC motors, it is essential to determine key system parameters from the response curve, including the delay time (L) and time constant (T)[10]. These parameters provide critical insights into the dynamic behavior of the system and serve as foundational inputs for controller design. By employing the equation of a straight line, the values of L and T can be accurately calculated from the response curve data[11].

As illustrated in Figure 1, the response curve presents two distinct points with coordinates: $X_1=0.03688$, $Y_1=0.5884$, and $X_2=0.3227$, $Y_2=6.791$. These points are strategically selected to derive a linear approximation of the system's initial response. The slope and intercept of the straight line are used to calculate the delay time and time constant, which are essential for subsequent control parameter tuning.

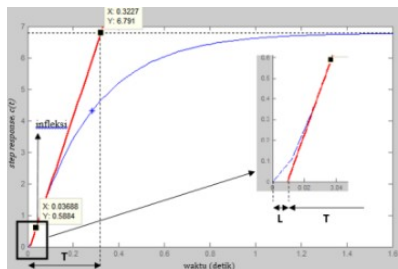


Figure 1. Parameter PID

General form of straight line equation

$$Y_2 - Y_1 = m(X_2 - X_1)$$

Where M is gradient slope of the line.

$$6.791 - 0.588 = m(0.3227 - 0.03688)$$

$$m = 21.701$$

The tangent line touches the x axis at a point with coordinates (X,0), then

$$Y_2 - 0 = m(X_2 - X)$$

$$6.791 - 0 = 21.701(0.3227 - X)$$

$$X = 0.009765$$

$$T = (X_2 - L)$$

$$T = 0.3227 - 0.009765 = 0.3130$$

After value L and T already know, Next step is find a PID Parameter

- Propotional (Kp):

$$K_p = 1.2 (T/L) = 1.2 (0.3130/0.009765) = 38.464$$

- Integral (Ki):

$$T_i = 2L, K_i = K_p/T_i = 38.464/2(0.009765) = 1969.483$$

- Derivative (Kd):

$$T_d = 0.5L, K_d = T_d K_p = 0.5(0.009765)(38.464) = 0.1878$$

3. LQR

To derive the Q and R matrices for the Linear Quadratic Regulator (LQR) controller. The script, developed based on the trial-and-error method[12]. This method involves iterative tuning to achieve optimal system performance while adhering to the mathematical requirements of the LQR design. Specifically, the Q matrix must be a real, positive semidefinite matrix ($Q \geq 0$), while the R matrix must be a real, positive definite matrix ($R > 0$)[13][14]. For the initial tuning, the Q matrix was set to:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and the R matrix was initialized as:

$$R = [1, 011]$$

These matrices play a crucial role in shaping the behavior of the LQR controller by influencing the weighting of state variables and control effort in the cost function[15][16]. The process of determining

the optimal values for these matrices involved the trial-and-error method, which was used to iteratively refine the matrices. This approach aimed to balance competing objectives, such as minimizing energy consumption while ensuring precise and stable control of the DC motor.

The MATLAB-based approach provided flexibility in exploring various combinations of Q and R values to achieve the desired dynamic performance[17][18]. By adjusting these matrices, different configurations were tested to fine-tune the system's behavior and responsiveness. This iterative process allowed for a more comprehensive evaluation of the controller's performance in different scenarios.

The final matrix values were selected based on a thorough analysis of system response metrics, such as settling time, overshoot, and steady-state error[19][20]. These metrics provided a clear indication of how well the system was performing and helped guide the selection of the optimal parameters. This ensured that the LQR controller could be fine-tuned to meet the desired performance criteria.

As a result, the LQR controller not only stabilized the DC motor system but also optimized performance across a variety of operating conditions. The chosen matrix values facilitated a well-rounded performance, ensuring that the system was both efficient and stable under different conditions.

III.RESULT & DISCUSION

1.PID Simulation

A. PID Simulation circuit in 700 rpm speed reference

The simulation process was carried out by setting a reference speed of 700 rpm, as shown in Figure 2, to evaluate the system's performance under controlled conditions. This reference speed was chosen to test the controller's ability to maintain a consistent and stable motor speed over time. To regulate the motor speed, a Proportional-Integral-Derivative (PID) controller was employed, which is one of

the most widely adopted control strategies in various applications, including motor speed regulation. The PID controller is known for its simplicity and effectiveness, making it an ideal choice for this type of system.

Throughout the simulation, the PID controller demonstrated its ability to maintain the target speed of 700 rpm with relatively minimal deviation, showing its effectiveness in motor speed regulation. The results of the simulation highlighted the controller's ability to stabilize the system and respond promptly to disturbances, making it a reliable choice for motor control in this context. The simplicity of the PID controller, combined with its proven track record, makes it an attractive option for many engineering applications requiring precise control of dynamic systems.

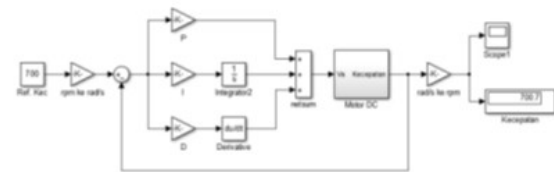


Figure 2: PID Controller Simulation at 700 RPM

The system successfully achieved a steady-state speed of 700.7 rpm, demonstrating the PID controller's capability to maintain a precise setpoint. The dynamic response of the rotor speed for the given reference input is illustrated in Figure 3, which highlights the time-domain performance of the system under the specified conditions.

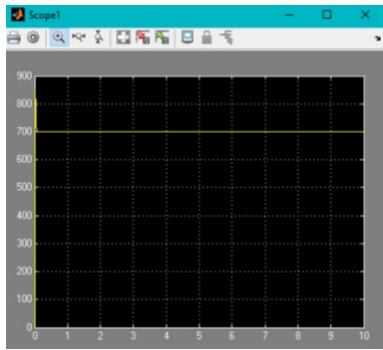


Figure 3: Rotor Speed Response with PID Controller at 700 RPM

The performance metrics extracted from Figure 3 provide insights into the controller's dynamic behavior:

- Rise time : 7.025 ms,
- Settling time : 53.7 ms,
- Max. Overshoot : 21.04 %
- Error steady state : 0.001 %

B. PID Simulation circuit in 1000 rpm speed reference

The simulation process was conducted by applying a reference speed of 1000 rpm, as depicted in Figure 4, to assess the system's performance under controlled conditions. This reference speed was selected to evaluate the controller's effectiveness in maintaining a stable and accurate motor speed over time. To regulate the motor speed, a Proportional-Integral-Derivative (PID) controller was utilized, a control strategy that is widely recognized for its simplicity and effectiveness in various applications, particularly in motor speed regulation. The PID controller is a popular choice due to its straightforward tuning process and well-established implementation practices.

Throughout the simulation, the PID controller demonstrated its ability to maintain the target speed of 1000 rpm with minimal deviation, highlighting its reliability in motor speed regulation. The results of the simulation showed that the controller effectively stabilized the system, responding

to disturbances promptly and ensuring consistent performance. This reaffirms the PID controller's ability to handle dynamic motor control tasks efficiently. The simplicity of the PID controller, coupled with its proven effectiveness, makes it an ideal solution for many engineering applications where precise control of motor speed is essential.

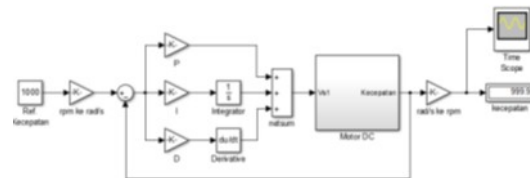


Figure 4: PID Controller Simulation at 1000 RPM

The system successfully achieved a steady-state speed of 1000 rpm, demonstrating the PID controller's capability to maintain a precise setpoint. The dynamic response of the rotor speed for the given reference input is illustrated in Figure 5, which highlights the time-domain performance of the system under the specified conditions.

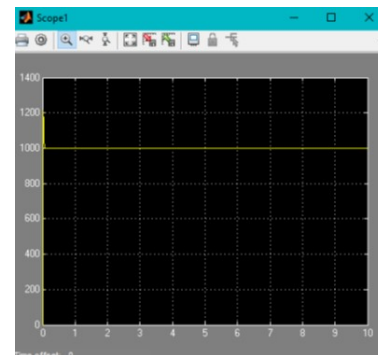


Figure 5: Rotor Speed Response with PID Controller at 1000 RPM

The performance metrics extracted from Figure 5 provide insights into the controller's dynamic behavior:

- Rise time : 6.910 ms
- Settling time : 53.5 ms
- Max. Overshoot : 21.08 %

- Error steady state : 0.0007 %

C. PID Simulation circuit in 1300 rpm speed reference

The simulation process was conducted by applying a reference speed of 1300 rpm, as depicted in Figure 6, to evaluate the system's performance under controlled conditions. This reference speed was chosen to test the controller's ability to maintain a precise and stable motor speed over time. To regulate the motor speed, a Proportional-Integral-Derivative (PID) controller was employed, a widely adopted control strategy known for its simplicity and effectiveness in various applications, particularly in motor speed regulation. The PID controller is preferred due to its easy tuning process and well-established implementation practices in engineering systems.

Throughout the simulation, the PID controller demonstrated its capacity to maintain the target speed of 1300 rpm with minimal deviation, showing its reliability in regulating motor speed. The results of the simulation highlighted the controller's effectiveness in stabilizing the system and responding promptly to any disturbances, ensuring consistent performance. The proven effectiveness of the PID controller, combined with its simplicity and straightforward implementation, makes it a reliable choice for motor control tasks in various dynamic systems that require precise and stable performance.

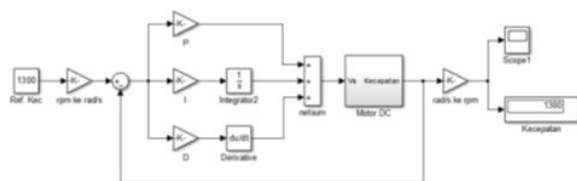


Figure 6: PID Controller Simulation at 1300 RPM

The system successfully achieved a steady-state speed of 1300 rpm, demonstrating the PID controller's capability to maintain a precise setpoint. The dynamic response of the rotor speed for the given reference input is illustrated in Figure 7, which highlights the time-domain performance of the system under the specified conditions.

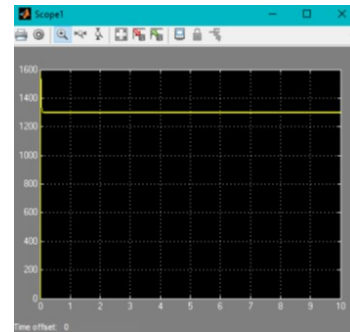


Figure 7: Rotor Speed Response with PID Controller at 1300 RPM

The performance metrics extracted from Figure 7 provide insights into the controller's dynamic behavior:

- Rise time : 6.995 ms
- Settling time : 53.7 ms
- Max. Overshoot : 21.09 %
- Error steady state : 0 %

2. LQR Simulation

A. LQR Simulation circuit in 700 rpm speed reference

The simulation for the Linear Quadratic Regulator (LQR) controller was performed with a reference speed set to 700 rpm, as shown in Figure 8. The LQR controller, known for its optimal control capabilities, was used to maintain the system's stability and minimize disturbances during operation.

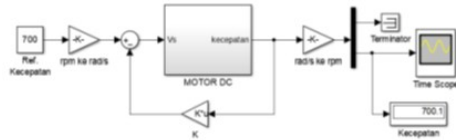


Figure 8: LQR Controller Simulation at 700 RPM

The system successfully reached a steady-state speed of precisely 700 rpm, highlighting the controller's ability to maintain the desired setpoint with minimal variation. This demonstrates the controller's effectiveness in stabilizing the system and ensuring reliable operation without significant deviation.

Figure 9 presents the rotor speed response to the reference input, visually depicting the system's performance under the regulation of the LQR controller. The graph illustrates how the controller maintains the rotor speed closely aligned with the target value throughout the process.

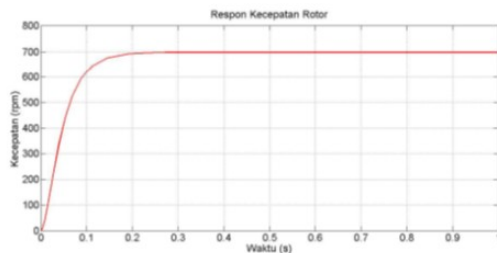


Figure 9: Rotor Speed Response with LQR Controller at 700 RPM

Key performance metrics obtained from the simulation, as shown in Figure 9, are as follows:

- Rise time : 91.406 ms
- Settling time : 171.2 ms
- Max. Overshoot : 0 %
- Error steady state : 0.00014 %

B. LQR Simulation circuit in 1000 rpm speed reference

The simulation for the Linear Quadratic Regulator (LQR) controller was performed with a reference speed set to 1000 rpm, as

shown in Figure 9. The LQR controller, known for its optimal control capabilities, was used to maintain the system's stability and minimize disturbances during operation.

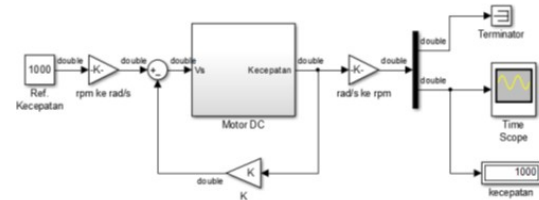


Figure 10: LQR Controller Simulation at 1000 RPM

The system successfully reached a steady-state speed of exactly 1000 rpm, demonstrating the controller's ability to maintain the desired setpoint with minimal deviation. This achievement reflects the controller's effectiveness in stabilizing the system and ensuring consistent performance.

Figure 11 illustrates the rotor speed response to the reference input, providing a clear depiction of the system's behavior when regulated by the LQR controller. The graph highlights how the controller effectively manages the rotor speed, keeping it closely aligned with the target value throughout the operation.

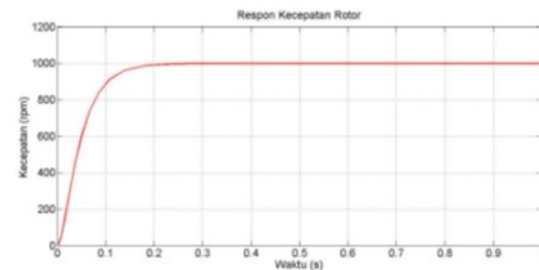


Figure 11: Rotor Speed Response with LQR Controller at 1000 RPM

Key performance metrics obtained from the simulation, as shown in Figure 11, are as follows:

- Rise time : 90.877 ms

- Settling time : 169.8 ms
- Max. Overshoot : 0 %
- Error steady state : 0 %

C. LQR Simulation circuit in 1300 rpm speed reference

The simulation for the Linear Quadratic Regulator (LQR) controller was performed with a reference speed set to 1300 rpm, as shown in Figure 12. The LQR controller, known for its optimal control capabilities, was used to maintain the system's stability and minimize disturbances during operation.

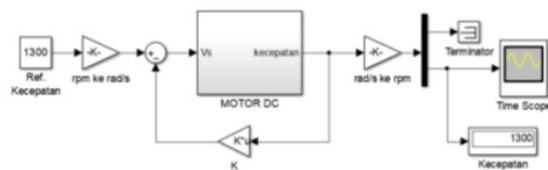


Figure 12: LQR Controller Simulation at 1300 RPM

The system successfully achieved a steady-state speed of precisely 1300 rpm, highlighting the effectiveness of the controller in maintaining the desired setpoint with minimal fluctuation. This consistent performance indicates that the control mechanism effectively stabilizes the rotor speed without significant deviation, ensuring reliable operation.

Figure 13 illustrates the rotor speed's response to the reference input, providing a visual representation of the system's performance under the regulation of the LQR controller. The graph emphasizes the controller's ability to keep the rotor speed closely aligned with the target value throughout the process.

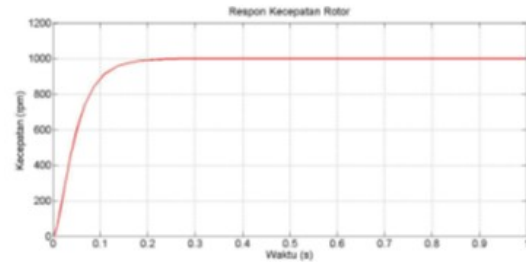


Figure 13 : Rotor Speed Response with LQR Controller at 1300 RPM

Key performance metrics obtained from the simulation, as shown in Figure 13, are as follows:

- Rise time : 89.743 ms
- Settling time : 166.9 ms
- Max. Overshoot : 0 %
- Error steady state : 0 %

IV.CONCLUSION

Based on the comprehensive simulation results conducted to compare the Proportional-Integral-Derivative (PID) and Linear Quadratic Regulator (LQR) controllers for DC motor speed regulation, the following conclusions can be drawn:

- The PID controller demonstrates a faster response in achieving a steady speed compared to the LQR controller. This is evident from the simulation data, where the rise time and settling time are considerably shorter for the PID controller. The PID's ability to reach a steady-state more quickly makes it suitable for applications where rapid speed adjustments are crucial.
- The rotor speed response with the LQR controller shows a distinct advantage in terms of stability. Unlike the PID-controlled system, the LQR controller exhibits zero overshoot, ensuring that the system stabilizes exactly at the reference speed. In contrast, the PID controller experiences a significant overshoot,

approximately 20%, before eventually reaching the steady-state. This indicates that while the PID controller provides quicker responses, it may introduce undesirable transients in certain applications where precision and stability are paramount.

- The results indicate that the LQR controller is better suited for applications where minimizing overshoot and ensuring precise control are the primary concerns. On the other hand, the PID controller is more appropriate in systems where quick response times are required, albeit at the expense of some stability.

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