

Analysis of Optimal Control Using the Linear Quadratic Regulator (LQR) Method in DC Motors: Relevance to Community Engagement Programs

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Abstract: *DC motors are indispensable across various industrial sectors due to their numerous advantages, including high torque output, absence of reactive power losses, non-disruptive impact on electrical supply harmonics, and superior control accuracy. In the realm of technological advancement, automatic control systems play a pivotal role in ensuring operational efficiency. A robust control system must adhere to predefined criteria, particularly those related to performance indices such as accuracy, stability, and response speed. Optimal control systems are designed based on the principle of performance index optimization, ensuring that system parameters are configured to achieve maximum or minimum values of desired operational metrics. In the context of DC motor speed regulation, the Linear Quadratic Regulator (LQR) optimal control technique offers a sophisticated approach by optimizing the performance index through precise calibration of the Q matrix. This process generates the K feedback amplifier matrix and the optimal L tracking matrix, which collectively enhance the motor's performance. This research emphasizes the integration of LQR in community engagement programs, demonstrating its potential for application in renewable energy systems, such as solar-powered water pumps or automated machinery for local industries. By bridging technical innovation with societal needs, the study highlights the role of advanced control techniques in fostering sustainable development and improving the quality of life in underserved communities.*

Keyword: *DC motor, modeling, optimization, Linear Quadratic Regulator, Linear Quadratic Tracking*

Introduction

Direct Current (DC) motors are among the most widely used types of electric motors across various industries due to their excellent regulation characteristics. These motors play a crucial role in many applications, as they effectively convert DC electrical energy into mechanical energy in the form of rotational motion[1][2]. To analyze a DC motor system, a mathematical model of its variables is necessary, typically expressed as transfer functions. These transfer functions, often of the first or second order, are simulated using tools like MATLAB Simulink to study the motor's behavior and optimize its performance.

The physical structure of a DC motor resembles that of a DC generator and consists of three main components: field windings located in the stator, armature windings situated in the rotor, and an air gap between the field and armature windings. This structure is fundamental to the motor's function, as it allows electrical energy to be converted into mechanical energy through the interaction of these components[3][4][5].

Control systems are designed to manage a system's output to produce desired responses effectively. To develop an accurate model of a control system, simulations are often used because they are more cost-

effective and efficient compared to direct operations on physical systems. These simulations provide engineers with valuable insights into the system's behavior and performance, allowing them to optimize the control design before real-world implementation. Automatic control systems have become an integral part of modern science and technology, offering significant advantages such as reducing monotonous human tasks and increasing production capacity[6][7]. In particular, optimal control systems can enhance performance and precision by minimizing deviations from ideal conditions. The optimization process is guided by performance indices that serve as benchmarks for evaluating the system's performance[8].

An effective control system ensures rapid, stable responses without excessive energy consumption. By adjusting the performance index, the system can be fine-tuned for optimal control[9]. Such systems are essential in various community engagement initiatives, such as renewable energy projects or the development of low-cost automation tools for local industries, as they ensure sustainable and efficient operation[10].

This study focuses on using DC motors as the plant system to be controlled with the Linear Quadratic Regulator (LQR) method. The optimization of the performance index is achieved by calibrating the Q matrix, which generates the feedback gain matrix K and the optimal tracking matrix L . These matrices are key to ensuring the optimal performance of the DC motor. Before applying the LQR method, the transfer function of the DC motor must first be determined and converted into state-space representation. This step is critical for accurately calculating

the LQR parameters, which are then used to achieve the desired system performance.

Integrating advanced control methods like LQR into community engagement programs has profound implications. For instance, DC motors optimized through LQR can be employed in community-driven renewable energy projects, such as solar-powered water pumps for irrigation or automated machinery in small-scale industries. These applications highlight how technical innovations can directly address societal needs, enhance productivity, and contribute to sustainable development, especially in underserved communities. By leveraging simulations and control design, this study not only explores technical solutions but also emphasizes their applicability in real-world, community-focused initiatives. The findings highlight the importance of aligning technological advancements with societal impact, fostering a collaborative approach to problem-solving in community engagement programs. Through such efforts, technology can improve lives and contribute to growth in communities, demonstrating the valuable connection between engineering advancements and social progress.

Methodology

1. Research map

The research process begins with identifying the DC motor model to be tested and analyzing the procedures for the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT)[11]. This involves selecting the weight matrices Q and R , which play a crucial role in determining the system's performance. Once the weight matrices are chosen, the feedback value K is calculated. This

calculation enables simulations to be conducted, allowing the closed-loop system response to be determined and analyzed to evaluate overall system performance.

The system's characteristics in this study are defined as the unique traits or performance specifications that describe how the system responds to a given input[12]. When an input signal is applied, the system generates an output response, which can be classified into two main types: time response characteristics and frequency response characteristics[13]. This research focuses on the time response characteristics of the DC motor, aiming to observe how the system's output response evolves over time. This observation provides critical insights into the system's stability, speed, and efficiency in reaching the desired state following disturbances or input signals.

By understanding and analyzing the time response characteristics, this study seeks to offer a comprehensive understanding of the DC motor's performance, particularly in the context of implementing LQR and LQT procedures for system control optimization[14]. The findings from this analysis will serve as a foundation for subsequent steps in developing more efficient and effective control models.

2. Mathematic model

A simple representation of a problem or phenomenon that occurs and is presented in mathematical concepts. After that, from the problems that occur, a problem will be formed. The existence of this

mathematical model makes it easier to solve a problem.

- LQR method
 - *Cost Function*

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt$$

Q = state weighting factor (positive semi-definite matrix)

R = Control variable factor weight (positive definite matrix)

u = control signal

optimal K value of the performance index

$$K = R^{-1} B^T P$$

Where the matrix P in the equation above must satisfy the reduced equation

$$A^T P + P A - P B R^{-1} B^T P + Q = 0$$

The transfer function of a DC motor is using the Ziggler Nichols Tuning method approach.

Laplace transform:

$$\tau s T(s) + T(s) = T i(s)$$

$$\frac{T(s)}{T i(s)} = \frac{1}{\tau s + 1}$$

Electric characteristic

$$\frac{di_a}{dt} = \frac{R}{L} i_a - \frac{K}{L} \omega_r + \frac{1}{L} V_a$$

$$\frac{d\omega_a}{dt} = \frac{K}{J} i_a - \frac{B}{J} \omega_r$$

Mechanical Characteristic

$$T = K i_a$$

State Space model

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega_a}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{di_a}{dt} & -\frac{K}{L} \\ \frac{K}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_a$$

$$= Ax + Bu$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} V_a$$

$$= Cx + Du$$

$$u = -Kx$$

Ordo 1 model

A first order system has a variable s with the highest power being one. The mathematical model of the first order system of a DC motor can be written as follows:

$$G(s) = \frac{K}{T + K}$$

$$K = \frac{T}{i}$$

Information :

K = coefficient

τ = DC motor torque

I = DC motor current

$$Ks = \frac{0,19}{5.1}$$

$$Ks = 0.372 \text{ Nm/A}$$

$$G(s) = \frac{0,0372}{0,19s + 0,0372}$$

3. DC Motor

Direct current motors (DC motors) are one of the types of electric motors most widely used in industry and in the future these machines will continue to be used because of their good regulatory characteristics[15]. The working principle is based on the interaction between two magnetic fluxes called the field coil and the armature coil. The form of energy produced will be in the form of rotation.

4. Linear Quadratic Regulator (LQR)

Linear Quadratic Regulator (LQR) is a method for designing modern constraint systems based on state space[16]. The LQR controller has two parameters, namely the weight matrix Q and R , which must be determined so that it can produce optimal control actions as expected. LQR controls the process/plant using a linear combination of the plant states. To design an LQR controller, the first step that must be taken is to choose a weight matrix of Q and R values[17]. The input R is heavier than the state while the state weight value is more than the input. Then the feedback K can be calculated and the closed loop response of the system can be found by simulation[18][19]. LQR controller design where the selection of the Q and R weight matrices is guided by the greater the value of Q , the closer it will be to the minimum point and the greater the value of R , the smaller/minimum the energy used[20].

5. Block Diagram

It is part of a diagram of a simulation process in which each component, part, or function of each process is depicted in a system of blocks connected by lines as a guide to the simulation. The block diagram contains input, process and output from the simulation process being run.

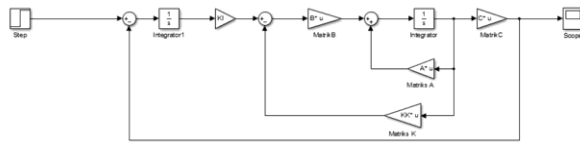


Figure 1. LQR block diagram

Results and Discussions

Table 1. specifications for DC motor type C23-L50

Performance data	
Rated voltage (V)	12 Vdc
Speed	1600 RPM
Average Torque	0.19
Reduction rate	22
Nominal current (A)	5.1 Amps

Table 2. Parameter for DC motor type C23-L50

Parameter Motor DC tipe C23-L50		
Parameter	Symbol	Size and unit
Inertial rotor	J_m	0.0000459 Kg.m ²
Damping	B	0.001 N.m/(rad/s)
Rated Torque	K_t	0.19 N.m/A
Back EMF	K_E	0.0519 V/(rad/s)
Resistance	R_T	0.63 Ohm
Induktance	L	0.00077 H

1. LQR circuit

- Without Noise

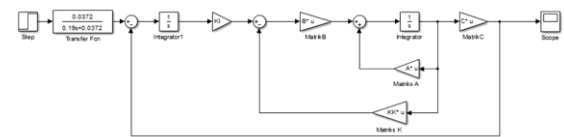


Figure 2. LQR circuit without noise

Figure 2 shows the LQR circuit with FCN transfer of order 1 obtained from the calculation above, the results on the scope can be seen in Figure 3

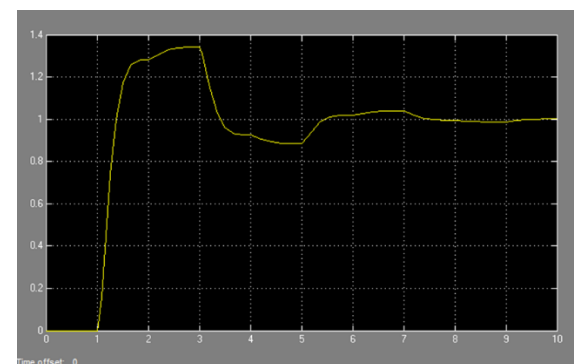


Figure 3. Waveform of LQR circuit without noise

The results in Figure 3 show that the DC motor used to test the optimal system with LQR is still not stable to order 1, as shown by the results of the overshoot that occurs and the stability value that does not occur immediately within a few seconds.

- With Noise

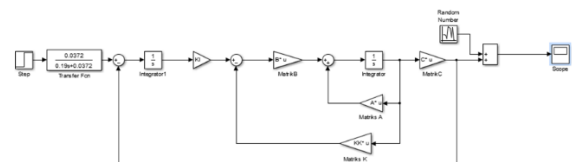


Figure 4. LQR circuit with noise

In figure 4 is an LQR circuit with added noise to find out how the circuit works if there is a problem

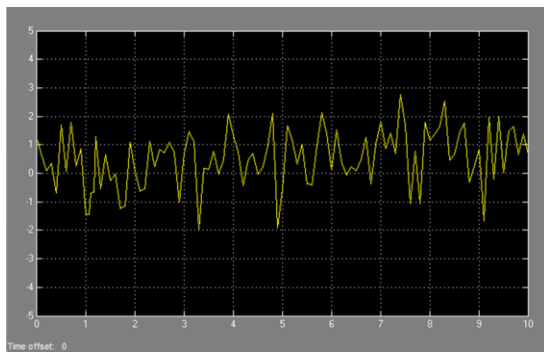


Figure 5. Waveform of LQR circuit with noise

The noise effect causes the waves in the system to fluctuate, which means the system cannot reach a stable condition or fixed point. Noise, which can be random interference or unwanted signals, affects the system's input and output signals, giving rise to unpredictable variations in its response. These fluctuations can result in difficulties in controlling the system precisely, because the signals received or processed do not represent actual conditions.

The impact of this noise can be very significant, especially in systems that require high stability, such as in DC motor control or other automated systems. Systems that are exposed to noise tend to experience oscillations, slow response, or even failure to reach the desired steady-state value. In more complex scenarios, noise can amplify errors in the control system, cause performance that is far from optimal, or even damage hardware if not handled properly.

Conclusion

Based on the results of simulation tests carried out using MATLAB, it can be concluded that the application of a Linear Quadratic Regulator (LQR) controller with order 1 has not been able to produce optimal

performance in the DC motor system being tested. The simulation results show that although the LQR controller has an influence on motor control, the system response obtained still shows quite significant fluctuations, which indicates a lack of stability in a relatively short time.

In this test, the DC motor system controlled with LQR order 1 did not succeed in reaching a stable condition quickly. Delays in reaching the desired fixed point as well as oscillations or fluctuations in the motor output indicate that the LQR controller with order 1 has not been able to optimize the motor speed or position settings in an efficient time. This indicates that although the LQR controller can reduce errors, its performance is not sufficient to provide a fast and stable response according to the needs of a more complex DC motor system.

Thus, this system cannot be said to be optimal because the resulting DC motor control still shows significant instability in a shorter time. To achieve optimal performance, further development is needed, such as increasing the order of the LQR controller or using other control methods that can reduce fluctuations and increase the stability of the DC motor system in a shorter time.

References

- [1] L. Ljung, "Prediction error estimation methods", *Circ. Syst. Signal Process.* 21 (2002) 11–21.
- [2] Tang, Wei-Jie, Zhen-Tao Liu, and Qian Wang. "Motor DC speed control based on sistem identification and PID auto tuning." *Control Conference (CCC), 2017 36th Chinese. IEEE, 2017.*

- [3] Ismeal, Godem A., Karol Kyslan, and Viliam Fedák. "Motor DC identification based on Recurrent Neural Networks." *Mechatronics- Mechatronika (ME)*, 2014 16th International Conference on. IEEE, 2014.
- [4] Kamdar, Sauyma, Harsidh Brahmbhatt, Tigmanshu Patel, and Manish Thakker. "Sensorless speed control of high speed brushed Motor DC by model identification and validation." In *Engineering (NUICONE)*, 2015 5th Nirma University International Conference on, pp. 1-6. IEEE, 2015.
- [5] Viola, J., L. Angel, and J. M. Sebastian. "Design and robust performance evaluation of a fractional order PID controller applied to a Motor DC ." *IEEE/CAA Journal of Automatica Sinica* 4, no. 2 (2017): 304-314
- [6] Wang X, Yin T, Chen J, et al. Rapid design of Motor DC speed control sistem based on Matlab[J]. *Applied Mechanics & Materials*, 2015, 743:168-171.
- [7] Basuki Arief. Pengaturan kecepatan Motor DC secara real ttime Menggunakan Teknik Kontrol Optimal Linear Quadratic Regulator (LQR). *Teknik Elektro UNDIP, Semarang*
- [8] Arrosida, Hanum. "Perancangan Metode LQR (Linear Quadratic Regulator) Sebagai Solusi Optimal Pengendalian Gerak Quadroter." *Seminar MASTER 2016*. Vol. 1. No. 1. 2016
- [9] Ogata, K. *Modern Control Engineering*, 4th edition, Prentice-hall, New Jersey (2002).
- [10] Nugraha, Anggara Trisna, and Rachma Prilian Eviningsih. *Konsep Dasar Elektronika Daya*. Deepublish, 2022.
- [11] Nugraha, Anggara Trisna, et al. "PEMODELAN DAN SIMULASI OPEN LOOP SERTA CLOSE LOOP MOTOR DC TYPE 18105 ORDE 2." *Prosiding Seminar Nasional Terapan Riset Inovatif (SENTRINOV)*. Vol. 10. No. 1. 2024.
- [12] Ivannuri, Fahmi, and Anggara Trisna Nugraha. "Implementation Of Fuzzy Logic On Turbine Ventilators As Renewable Energy." *Journal of Electronics, Electromedical Engineering, and Medical Informatics* 4.3 (2022): 178-182.
- [13] Achmad, Irgi, and Anggara Trisna Nugraha. "Implementation of Voltage Stabilizers on Solar Cell System Using Buck-Boost Converter." *Journal of Electronics, Electromedical Engineering, and Medical Informatics* 4.3 (2022): 154-160.
- [14] Jamil, M. H., et al. "The existence of rice fields in Makassar City." *IOP Conference Series: Earth and Environmental Science*. Vol. 681. No. 1. IOP Publishing, 2021.
- [15] Arifuddin, Rahman, et al. "Baby Room Temperature and Humidity Control System Using Fuzzy Logic." *Emitor: Jurnal Teknik Elektro* (2024): 275-280.
- [16] Sheila, Sindy Yurisma, et al. "Coffee Drying Tool with LQR-PID Control." *MEIN: Journal of Mechanical, Electrical & Industrial Technology* 1.2 (2024): 18-23.
- [17] Paluga, Ary Pratama, et al. "Renewable Energy System Optimization: Mppt Inverter Integration, Energy Storage

Systems, And Its Impact on Sustainability and Efficiency Use Of Energy." MEIN: Journal of Mechanical, Electrical & Industrial Technology 1.2 (2024): 12-17.

- [18] Hidayana, Elmi, et al. "UTILIZATION OF BAGASSE WASTE FOR BIO-OIL EXTRACTION FROM SUGARCANE JUICE VENDORS IN BANYUWANGI REGENCY." Jurnal Cakrawala Maritim 7.1 (2024): 43-51.
- [19] Rahman, Farhan Wahyu Nur, et al. "ASSESSING THE MARKETING APPROACH OF PT PAL INDONESIA COMPANY THROUGH A SWOT ANALYSIS." Jurnal Cakrawala Maritim 6.1 (2023): 20-24.
- [20] Handandi, Zukhruf Zidane, et al. "HOW THE PERFORMANCE OF A DC MOTOR SYSTEM IS IMPACTED BY NOISE." Jurnal Teknologi Maritim 6.1 (2023): 11-16.