

Position Control of a DC Motor Using PID and LQR Methods

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Abstract

In this study, a control system is developed for positioning a DC motor using optimal control methods to improve performance in terms of settling time, rise time, and the design of feedback gain K to minimize the cost function. The control approaches implemented include the Proportional-Integral-Derivative (PID) method and the Linear Quadratic Regulator (LQR) algorithm. The PID controller is designed to manage the motor's position by calculating the gain K using the Routh-Hurwitz criterion. The specific parameters K_p , K_i , and K_d are then determined using the Ziegler-Nichols (Z-N) Oscillation method. For the LQR approach, motor position control is achieved by tuning the values of matrices Q and R , which influence the resulting $KLQR$ gain. Simulations are conducted using MATLAB Simulink to evaluate performance. At the conclusion of the study, the identified PID controller parameters are $K_p=3.72$, $K_i=1.2031$, and $K_d=2.8751$. For the LQR controller, the optimal Q matrix is $[100000;0100;000]$ with an R value of 0.01 .

Keywords: DC motor, proportional–integral–derivative (PID), Linear Quadratic Regulator (LQR)

1. Introduction

In addressing a problem, it is highly beneficial if the solution is simple yet precisely applied (Nugraha et al., 2023). Therefore, learning a control system is valuable, as its applications extend beyond just hardware and other aspects of regulation systems (Paluga et al., 2024). With the rapid development of technology today, control systems play a vital role in helping humans tackle various challenges (Saputra et al., 2024).

To implement such control systems digitally, it is essential to understand the fundamentals of control systems, as this knowledge greatly aids in applying these systems to industrial settings (Nugraha & Eviningsih, 2022). For this purpose, a controller is needed to maintain the stability of the motor's position so that it aligns with the desired target.

The application of optimal control theory for managing linear systems is widely utilized in both industry and education (Dermawan et al., 2023) (Aгна et al., 2023). Optimal control minimizes the energy function used, resulting in an optimal performance index (Sasongko et al., 2023). In this study, the control methods applied are the Proportional-Integral-Derivative (PID) Method and the Linear Quadratic Regulator (LQR) Algorithm (Nugraha & Ravi, 2023). By optimizing with these PID and LQR methods, a stable position for the DC motor can be achieved.

2. Material and methods

2.1. Material

2.1.1. DC Motor

A DC motor is a device that converts electrical energy into kinetic energy or motion. This type of motor, also called a Direct Current (DC) Motor, operates through the application of a steady voltage, which generates a continuous flow of current in one direction (Satrionata et al., 2023). This current creates a magnetic field that interacts with the motor's internal components, resulting in rotational motion. DC motors are widely used in various applications due to their simple control, ease of use, and reliability.



Figure 1. DC Motor

2.2. Methods

2.2.1. PID Controller

PID Controller (from the English abbreviation: Proportional-Integral-Derivative controller) is a feedback control mechanism commonly used in industrial control systems. A PID controller continuously calculates the error value as the difference between the desired setpoint and the measured process variable (Prastyawan & Nugraha, 2022). The controller attempts to minimize this error value at all times by adjusting control variables, such as the position of a control valve, damper, or power to a heating element, to a new value (Nguyen et al., 2023). The coefficients for the proportional, integral, and derivative terms, respectively (or P, I, and D), are used in this model.

- P is responsible for the current error value. For example, if the error value is large and positive, the control output will also be large and positive.
- I is responsible for the accumulated error over time. For instance, if the current output is smaller, the error will continue to accumulate, and the controller will respond with a higher output.
- D is responsible for predicting the future error value, based on the rate of change over time.

Because the PID controller only relies on the measured process variable, not on knowledge of the process itself, it can be widely used. By adjusting (tuning) the three model parameters, the PID controller can meet the process requirements. The controller's response can be explained by how it reacts to the error, the magnitude of overshoot from the setpoint, and the degree of system oscillation (Kawakibi, 2022). The use of algorithm does not guarantee optimal system control or even its stability (Ndruru, 2022). Some applications might use only one or two terms to provide an adequate system control. This can be achieved by setting the other parameters to zero. A PID controller can become a PI, PD, P, or I controller depending on which action is used. The PI controller is typically the most common controller.

2.2.2. LQR Linear Quadratic Regulator (LQR)

The Linear Quadratic Regulator (LQR) is an optimal control method for systems based on state space (Taherian et al., 2021) (Hilda, 2023). The LQR controller has two parameters, which are the weight matrices Q and R, that must be determined to produce an optimal control action as desired (Zhao et al., 2023) (Arrofiq et al., 2021). Unlike the Proportional-Integral-Derivative (PID) controller, which has tuning methods based on systematic approaches such as Ziegler-Nichols and Cohen-Coon, the LQR controller does not have a specific systematic tuning method for determining the weight matrices Q and R (Firdaus et al., 2023) (Susanto et al., 2020).

2.2.3. Mathematical Modeling of a DC Motor

Since this device is an implementation of the DC motor's position control, a DC motor circuit is used. To perform the modeling, there are two parts that will be analyzed in this circuit: the electrical system and the overall DC motor system. Combining both the electrical and mechanical equations, we can derive the transfer function of the overall DC motor system.

$$G(s) = \frac{10k}{30s^3 + 60s^2 + 31s}$$

2.2.4. Desain PID

To calculate the Gain Margin and Frequency Margin using the Routh-Hurwitz method (Zein, 2021). This process will provide insights into the system's stability and the limits of how much gain or frequency can be applied before the system becomes unstable. The results obtained will then be displayed to give a clearer picture of the system's stability margins. $K_c = 6,2$; $K_p = 6,18$

Table 1. Parameter controller

	Kp	Tr	Td
P	0,5 Kc		
PI	0,4 Kc	Pc/1,2	
PID	0,6 Kc	0,5Pc	Pc/8

- Controller P
 $G_c(s) = Kp$
 $G_c(s) = 3,1$

- Controler PI

$$G_c(s) = Kp \left(1 + \frac{1}{Tr \cdot s} \right)$$

$$G_c(s) = 2,79 + \frac{0,541}{s}$$

- Controler PID

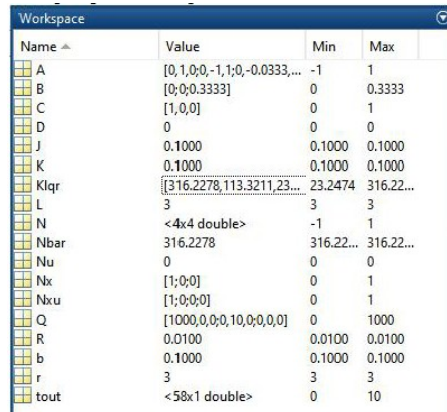
$$G_c(s) = Kp \left(1 + \frac{1}{Tr \cdot s} + Td \cdot s \right)$$

$$G_c(s) = 3,72 + \frac{1,2031}{s} + 2,8751 s$$

3. Results and discussion

3.1. LQR Controler

The simulation is conducted using MATLAB. The values of the parameters to be used will be saved in the workspace.



Name	Value	Min	Max
A	[0, 1, 0, 0, -1, 1, 0, -0.0333, ...]	-1	1
B	[0; 0; 0.3333]	0	0.3333
C	[1, 0, 0]	0	1
D	0	0	0
J	0.1000	0.1000	0.1000
K	0.1000	0.1000	0.1000
Klqr	[[316.2278, 113.3211, 23.2474, ...]]	23.2474	316.22...
L	3	3	3
N	<4x4 double>	-1	1
Nbar	316.2278	316.22...	316.22...
Nu	0	0	0
Nx	[1; 0; 0]	0	1
Nxu	[1; 0; 0; 0]	0	1
Q	[1000, 0, 0, 0, 10, 0, 0, 0]	0	1000
R	0.0100	0.0100	0.0100
b	0.1000	0.1000	0.1000
r	3	3	3
tout	<58x1 double>	0	10

Figure 2. Workspace

Figure 2 showing the value of parameter to be used in this research. After knowing the base value parameter to be use, the next step is make a modeling of a DC motor using SIMULINK.

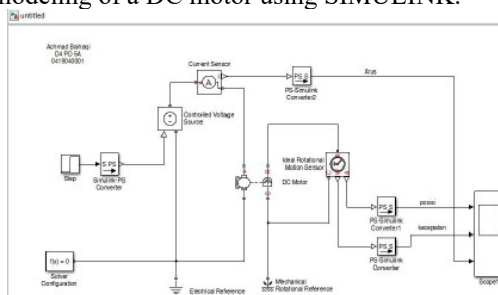


Figure 3. Modeling DC Motor

Following the simulation, the comparison of the system's response graph to the input signal is shown below.

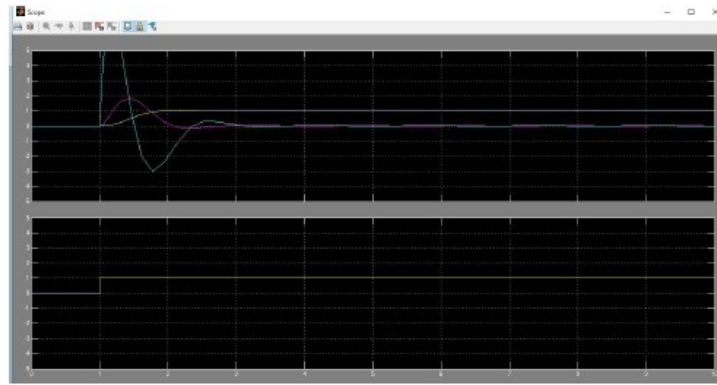


Figure 4. comparison response

The graph shows that the system responds quickly and achieves stability in under 2 seconds. After knowing the respond of the graph, adjust more variations of matrix Q and matrix R. Here is the result of type variations.

- Type 1 : $Q = [1000 \ 0 \ 0 ; 0 \ 10 \ 0 ; 0 \ 0 \ 10]$;

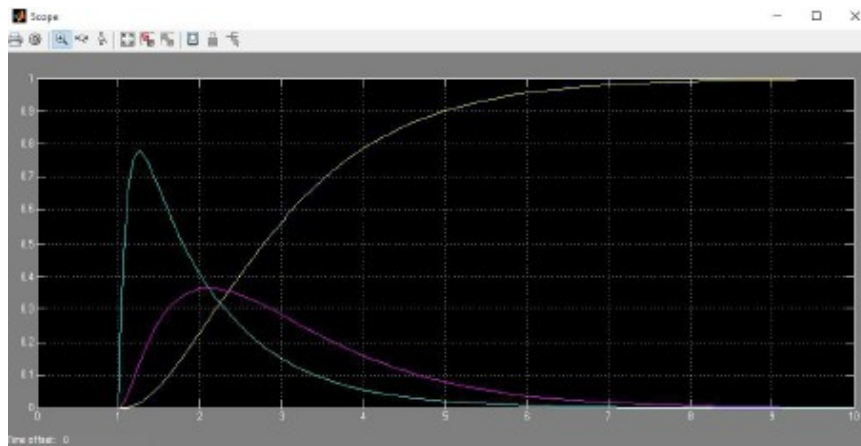


Figure 5. Respond of variation matrix Q type 1

In the graph, the resulting response indicates that as time progresses, the system's response becomes slower and more gradual. This could be due to a decrease in the system's bandwidth or an increase in damping, leading to a longer settling time and reduced system speed. A slower response can also imply that the system is approaching a more stable state, where the oscillations diminish and the system stabilizes at a steady value. This behavior is often observed when the system is under higher damping conditions or when the gain margin is reduced.

- Type 2: $Q = [1000 \ 0 \ 0 ; 0 \ 100 \ 0 ; 0 \ 0 \ 10]$;

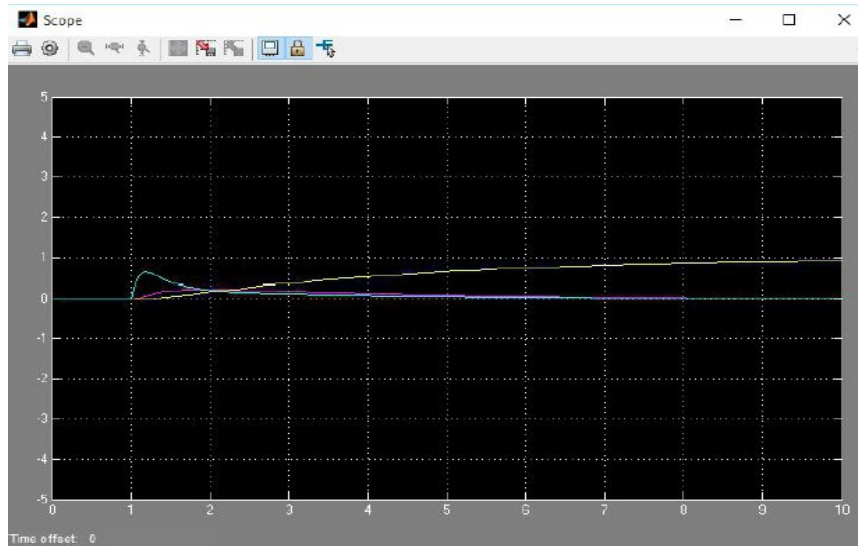


Figure 6. Respond of variation matrix Q type 2

In the graph, the resulting response indicates that the support system's performance deteriorates over time, failing to reach stability even after 10 seconds. This suggests that the system is experiencing instability or excessive oscillations, which prevent it from settling to a steady state. The failure to stabilize could be due to an inadequate gain margin, improper tuning of system parameters, or the presence of dominant unstable poles in the system's transfer function. As a result, the system might be underdamped or exhibit growing oscillations, making it unable to achieve a stable equilibrium within the expected timeframe.

- Type 3: $R = 1$;

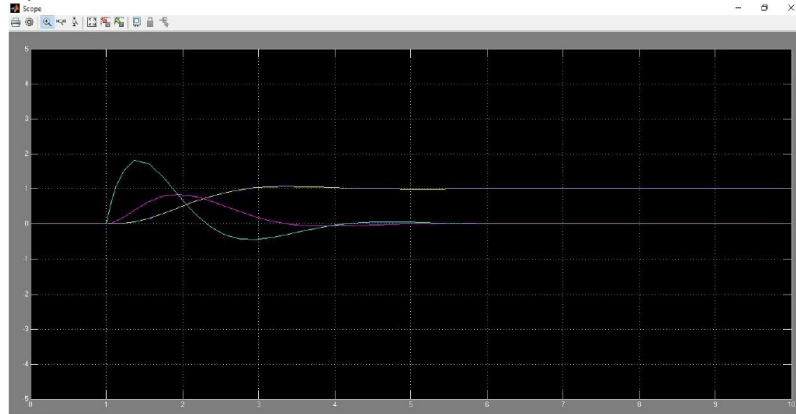


Figure 7. Respond of variation matrix R type 3

From the graph, it can be understood that the system's response is slow when using a value of $r = 0.01$. This indicates that the system takes a longer time to reach its steady-state or desired output. A smaller value of r may result in a more sluggish response, suggesting that the system's dynamics might be heavily damped or that the control parameters need adjustment to achieve faster response times.

4. Conclusion

- To control the position of the DC motor as desired using the Proportional-Integral-Derivative (PID) method, the optimal parameters for the PID controller were determined as $K_p = 3.72$, $K_i = 1.2031$, and $K_d = 2.8751$. With these settings, the system achieves stability at around 28 seconds.

- Using the Linear Quadratic Regulator LQR method, the best system response was achieved with a Q matrix of $[1000 \ 0 \ 0; \ 0 \ 10 \ 0; \ 0 \ 0 \ 0]$ and an R value of 0.01. With these values, the system responds quickly and reaches stability in under 2 seconds.

Credit authorship contribution statement

Author Name: Conceptualization, Writing – review & editing. **Author Name:** Supervision, Writing – review & editing. **Author Name:** Conceptualization, Supervision, Writing – review & editing.

References

- Nugraha, Anggara Trisna, et al. "Three Phase Controlled Rectifier Circuit for Characteristic Control in DC Shunt Motor." (2023).
- Paluga, Ary Pratama, et al. "Renewable Energy System Optimization: Mppt Inverter Integration, Energy Storage Systems, And Its Impact on Sustainability and Efficiency Use Of Energy." MEIN: Journal of Mechanical, Electrical & Industrial Technology 1.2 (2024): 12-17.
- Saputra, Fahmi Yahya, et al. "Efficiency Of Generator Set On Changes In Electrical Load On Fishery Vessels." MEIN: Journal of Mechanical, Electrical & Industrial Technology 1.2 (2024): 1-4.
- Nugraha, Anggara Trisna, and Rachma Prilian Eviningsih. Konsep Dasar Elektronika Daya. Deepublish, 2022
- Dermawan, Denny, et al. "Penyearah Setengah Gelombang 3 Fasa Tak Terkendali dari Generator AC 3 Fasa." Seminar MASTER PPNS. Vol. 8. No. 1. 2023.
- Agna, Diego Ilham Yoga, Rama Arya Sobhita, and Anggara Trisna Nugraha. "Penyearah Gelombang Penuh 3 Fasa Tak Terkendali dari Generator Kapal AC 3 Fasa." Seminar MASTER PPNS. Vol. 8. No. 1. 2023.
- Sasongko, Adhy, et al. "Estimation of the thrust coefficient of a Quadcopter Propeller using Computational Fluid Dynamics."
- Nugraha, Anggara Trisna, and Alwy Muhammad Ravi. "Experimental Study of the Effect of Excitation Current on the Output Voltage of a Self-excited Synchronous Generator." (2023).
- Satrianata, Lugas Jagad, et al. "EXAMINING OPTIMAL CONTROL THROUGH THE APPLICATION OF (LQR) AND (LQT) APPROACHES IN THE CONTEXT OF DC MOTORS." Jurnal Teknologi Maritim.
- Prastyawan, Rikat Eka, and Anggara Trisna Nugraha. "PENERAPAN TEKNOLOGI INFORMASI UNTUK PEMBELAJARAN TEST OF ENGLISH FOR INTERNATIONAL COMMUNICATION PREPARATION." Jurnal Cakrawala Maritim 5.1 (2022): 4-8.6.1 (2023): 17-22
- Nguyen, V. D., Vo, M. T., Tran, M. D., Dang, Q. D., Nguyen, T. D., & Nguyen, T. V. (2023). Trajectory Tracking and Stabilization Control of Rotary Inverted Pendulum based on LQR and LQT Techniques: Simulation and Experiment. Journal of Technical Education Science, 18(1), 1-11.
- Taherian, S., Halder, K., Dixit, S., & Fallah, S. (2021). Autonomous collision avoidance using MPC with LQR-based weight transformation. Sensors, 21(13), 4296.
- Zhao, J., Yang, C., Gao, W., & Park, J. H. (2023). Optimal Dynamic Controller Design for Linear Quadratic Tracking Problems. IEEE Transactions on Automatic Control.
- Firdaus, M. R., Berbudi, T. A., Nurrahma, S., Izzaulhaq, G., & Hudati, I. (2023). Identifikasi Sistem Motor DC dan Penerapan Kendali PID, LQR, dan Servo Tipe 1 Berbasis Arduino-MATLAB. Jurnal Listrik, Instrumentasi, Dan Elektronika Terapan, 4(1).
- Zein, A. N. (2021). Analisis Penerapan Kontroler Linear Quadratic Regulator (LQR) Tuning PID pada Pengendalian Motor Induksi 3 Fasa dengan Menggunakan Metode Direct Torque Control (DTC) (Doctoral dissertation, Institut Teknologi Kalimantan).
- Susanto, T., Riskiono, S. D., Rikendry, R., & Nurkholis, A. (2020). Implementasi Kendali LQR Untuk pengendalian sikap longitudinal pesawat flying wing. Electro Luceat, 6(2), 245-254.
- KAWAKIBI, M. M. (2022). Sistem Kendali Kecepatan Jalan Robot Humanoid Ketika Berjalan Lurus Menggunakan LQR (Doctoral dissertation, Universitas Gadjah Mada).
- Ndruru, F. A. R. (2022). Optimal Linear Quadratic Regulator Menggunakan Hybrid Particle Swarm Optimization-Differential Evolution Algorithm untuk Menstabilkan Frekuensi pada Single Machine Infinite Bus.
- Hilda, H. Optimalisasi Kecepatan Sistem Aliran Fluida Metoda Linear Quadratic Regulator. JEPIN (Jurnal Edukasi dan Penelitian Informatika), 9(1), 1-6.
- ARROFIQ, M., NUGROHO, L. S., FAHMIZAL, F., & APRIASKAR, E. (2021). Sistem Kendali Eddy Current Brakes Dinamometer menggunakan Linear Quadratic Regulator (LQR). ELKOMIKA: Jurnal Teknik Energi Elektrik, Teknik Telekomunikasi, & Teknik Elektronika, 9(4), 923.