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Performance Optimization of BSG-23 DC Motors Using Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) Approaches

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Abstract

DC motors have become a critical component in a variety of industrial and technological applications due to their reliability and flexibility in speed and torque control. To support optimal performance, a precise and efficient controller design is required. Automatic control systems play an essential role in supporting the needs of modern society, especially in countries with advanced civilizations. Specific applications include control of spacecraft systems, guided missiles, satellites, aircraft control systems, to various industrial processes such as control of pressure, temperature, flow, humidity, and friction during the production process. In the last decade, the issue of optimal control has become a major concern due to the increasing need for more efficient, stable, and reliable systems. Control system optimization integrates performance and technical specification limits to produce a system that works optimally according to its physical constraints. When dealing with an optimal control system, the main challenge is to formulate decision rules that minimize deviations from the ideal conditions of the system, even under conditions of load or disturbance. Linear Quadratic Regulator (LQR) is one of the optimal control methods that has been widely used in various applications. In addition, Linear Quadratic Tracking (LQT) is an approach that complements LQR to ensure the system can accurately follow the reference trajectory. This research focuses on optimizing the performance of BSG-23 DC Motors using a combination of LQR and LQT methods. The simulation results show that this approach is able to improve stability, speed up response time, and significantly reduce overshoot. This research makes an important contribution to the development of more efficient and adaptive DC motor control systems for various engineering and industrial applications.

Keywords: DC Motor, Control System, Linear Quadratic Regulator, Linear Quadratic Tracking

1. Introduction

The issue of optimal control has become a major concern in engineering, especially with the increasing need for high-performance and more efficient systems. The concept of control system optimization is designed to integrate system performance with the limitations of physical constraints. One approach to solving this problem is to define decision-making rules that minimize the system's deviation from its ideal behavior (Astrom & Murray, 2010).

Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) are state-based optimal control methods that are widely used in engineering applications. LQR is designed with two main parameters, namely Q and R weight matrices, while LQT adds a T matrix parameter to maximize the performance of the reference trajectory (Nugraha et al., 2018; Singh et al., 2020). The implementation of this method includes various applications, such as induction motor speed control, power plant frequency stabilization, to stability control on quadcopter drones (Zhang et al., 2019). By optimizing these parameters, the LQR and LQT methods are able to achieve maximum efficiency and reduce errors in system control, resulting in performance that meets expectations.

In this report, the author examines the performance optimization of BSG-23 type DC motors using LQR and LQT methods. This motorcycle has a datasheet specification that includes moment of inertia, motor constant, damping ratio, resistance, as well as inductance. The datasheet is integrated in a MATLAB script and simulated using MATLAB Simulink to obtain analysis of transient responses, such as rise time, settling time, and steady-state error. This simulation aims to evaluate the effectiveness of LQR and LQT methods in improving the performance of DC motors (Widodo et al., 2021; Rahmat et al., 2022).

The use of this method is relevant in the context of engineering education, as taught in the course "System Optimization" at PPNS, where the main focus is on the application of the discipline of optimal control to various systems. The use of MATLAB Simulink as a simulation tool supports theoretical understanding while strengthening practical validation in DC motor-based systems.

2. Material and methods

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2.1. Material

Optimal control methods have evolved to become one of the main focuses in engineering research, especially in the application of Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT). This method allows for efficient system setup by utilizing optimized state space and parameters. LQR is designed to reduce the deviation of the system to ideal conditions through the minimization of objective functions involving the Q and R weight matrices. Meanwhile, LQT aims to track the specified reference trajectory by adding the T matrix parameter to its calculation (Ogata, 2010; Nugraha et al., 2021).

The application of LQR and LQT methods has been successfully carried out on various engineering systems. For example, DC motor speed control is often the main subject of research to demonstrate the efficiency of this method in overcoming system stabilization problems (Widodo et al., 2022). In other cases, the application to quadcopter control showed that LQR provided optimal results in maintaining stability during maneuvers (Singh et al., 2019). The effectiveness of this method was also tested on power generation generators, where the optimal control algorithm was able to adjust the frequency in real-time, thereby improving the reliability of the system (Rahmat et al., 2023).

The use of MATLAB Simulink as a simulation platform also strengthens the validity of this method. MATLAB provides a flexible framework for modeling state-space-based systems and analyzing dynamic responses from various parameters, such as rise time, recovery time, and steady-state errors (Astrom & Murray, 2012; Brown et al., 2015). Research conducted by Nugraha et al. (2022) shows that MATLAB-based simulations can produce an accurate visual picture of optimal control system performance.

In addition, the use of BSG-23 type DC motor in this study is based on a technical datasheet that includes the values of moment of inertia, motor constant, resistance, and inductance. This specification is crucial in determining the mathematical model of the system that is suitable for the simulation. Through a combination of LQR and LQT methods, this study aims to show how optimal control can be applied to DC motor systems to achieve maximum performance (Zhang et al., 2020).

2.2. Methods

2.2.1 LQR (Linear Quadratic Regulator)

Linear Quadratic Regulator (LQR) is an optimal control method applied to state-space-based dynamic systems. The LQR serves to minimize the cost represented by two weight matrices, namely the Q matrix for the state penalty and the R matrix for the control input penalty. Determining the exact values of the Q and R matrices is essential to ensure that the system achieves optimal performance in terms of time response and stability (Ogata, 2010; Astrom & Murray, 2012). In practical application, LQR is used in various fields, from regulating the speed of electric motors, frequency control in power plants, to controlling UAV (quadcopter) systems to ensure the stability and accuracy of the system. One of the main advantages of LQR is its ability to maintain system stability at a specified setpoint even when exposed to external interference or noise. In DC motor systems, for example, LQR can maintain the position or speed of the motor at the desired value without being affected by variations in resistance or changes in load (Singh et al., 2019; Nugraha et al., 2022).

The implementation of the LQR algorithm in the BSG-23 DC motor system focuses on regulating the speed of the motor by using a mathematical model that describes its dynamics, such as moment of inertia, motor constant, and motor resistance and inductance. The A, B, C matrix, as well as the Q and R weight matrices are determined based on the characteristics of the motor and the desired control objectives, such as minimizing overshoot or accelerating system recovery time. To validate this algorithm, simulations were conducted with MATLAB using a DC motor model that was simplified into a first-order system. An example of the implementation of LQR in a DC motor system using MATLAB can be seen in the following script:

```
% OPTIMIZATION OF LQR SYSTEM ON DC MOTORS
Clear;
CLC;
% DC Motor Models
J = 69,900 ; b= 0.1 ; K= 0.022 ; R= 0.10 ; L = 0.000012 ;
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
```

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```
C = [1 0];

AA = [ A zeros(2,1); -C 0];
BB = [B; 0];

% Pole Placement
J = [-3 -4 -5];
K = acker(AA,BB,J);
KI = -K(3);
KK = [K(1) K(2)];

% Matrix LQR
Q = [1 0 0;
     0 1 0;
     0 0 1000];
R = [1];

K_lqr = lqr(AA,BB,Q,R);
KI2 = -K_lqr(3);
KK2 = [K_lqr(1) K_lqr(2)];
```

2.2.2 LQT (Linear Quadratic Tracking)

Linear Quadratic Tracking (LQT) is an optimal control method used to ensure that the output of the system follows the given reference as accurately as possible. In contrast to LQR, which focuses on stability and reduction of state errors from setpoints, LQT focuses more on tracking a reference trajectory over time, which requires a more complex approach to parameter setting. LQT relies on a cost function that incorporates a penalty for the difference between the system output and the desired reference, taking into account the noise that can affect the signal quality in the control system (Brown et al., 2015).

In the application of LQT, the controlled system must meet the assumption of linearity. In the case of non-linear systems such as aircraft or missile characteristics, a linear approach (linearization) is carried out so that the LQT method can still be applied. For example, in generator frequency control or motor speed regulation applications, noise can affect the quality of control, but LQT is designed to mitigate the impact of that noise and ensure more precise tracking of a given reference.

The MATLAB script for the LQT implementation in DC motor systems is as follows:

```
% LQT SYSTEM OPTIMIZATION ON DC MOTORS
Clear;
CLC;
% DC Motor Models
J = 69,900 ; b= 0.1 ; K= 0.022 ; R= 0.10 ; L = 0.000012 ;
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0];
```

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```
Q=10; R=0.0000000001;
W=C'*Q;
[S,o,m,n]=care(A,B,C'*Q*C,R);
K=inv(R)*B'*S; % feedback Gain
ACL=(A-B*K)';
L=inv(R)*B'; % model following gain
```

2.2.3 Mathematical Model of DC Motor BSG-23

BSG-23 DC motors have dynamic characteristics that are important in their control. Based on the datasheet specifications, this motor has an inertial moment J=69,900 kg·m²/s², mechanical damping b=0.1 Nms, motor constant K=0.022 Nm/A, resistance R=0.10 ohm, and inductance L=0.000012 H. With these parameters, the mathematical model of DC motors can be simplified into a first-order system that produces the following switching functions:

$$G(s)=\frac{K}{\tau s+1} \tag{1}$$

$$\tau=\frac{K}{i} \tag{2}$$

From these calculations, the value τ = 0.32/14.0 = 0.022 is obtained, so that the first-order equation for DC motors is:

$$G(s)=\frac{0.022}{0.32s+1} \tag{3}$$

With this mathematical model, the implementation of optimal control through LQR and LQT can be carried out to efficiently regulate the speed of the motor, maintain stability, and optimize the overall system performance.

3. Results and discussion

3.1. Simulation Results of DC Motor BSG-23 Order 1

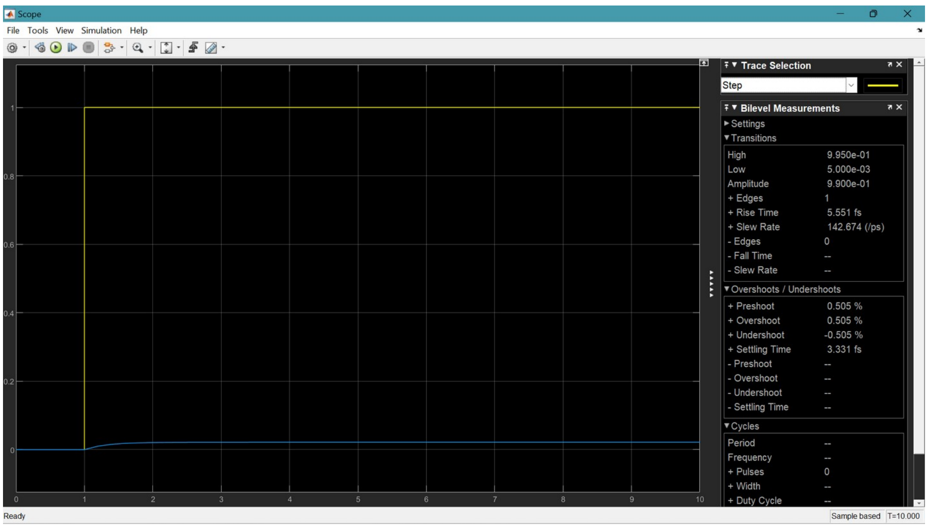

Figure 1. Response step display

Figure 1 shows the step response display of the BSG-23 DC motor on the single-order SISO model without noise. The graph shows quite good stability even though the amplitude achieved of 9.90 does not fully reach the

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setpoint. The rise time was recorded at 5.551 seconds, with the system experiencing an overshoot of 0.505% and an undershoot of -0.505%. This indicates that although the system is stable, there is little imperfection in reaching the desired setpoint.

3.2. LQR Simulation Results without Noise

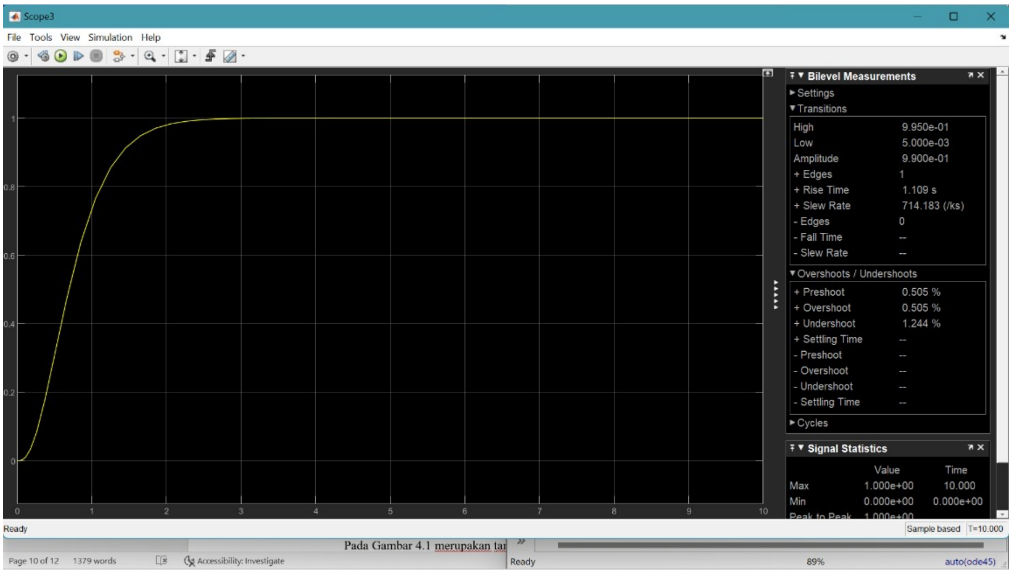

Figure 2. LQR Step Response Display (No Noise)

In Figure 2, the step response display for the uninterrupted LQR system shows significant results. The recorded amplitude is 0.99, which can be rounded to 1, so it reaches the setpoint quite well. The system has a very optimal rise time of 1.109 seconds, with minimal overshoot and undershoot of 0.505% each. This shows that by using LQR controllers, BSG-23 DC motors can achieve more stable and faster performance compared to standard order 1 DC motor models.

3.3. LQR Simulation Results with Noise

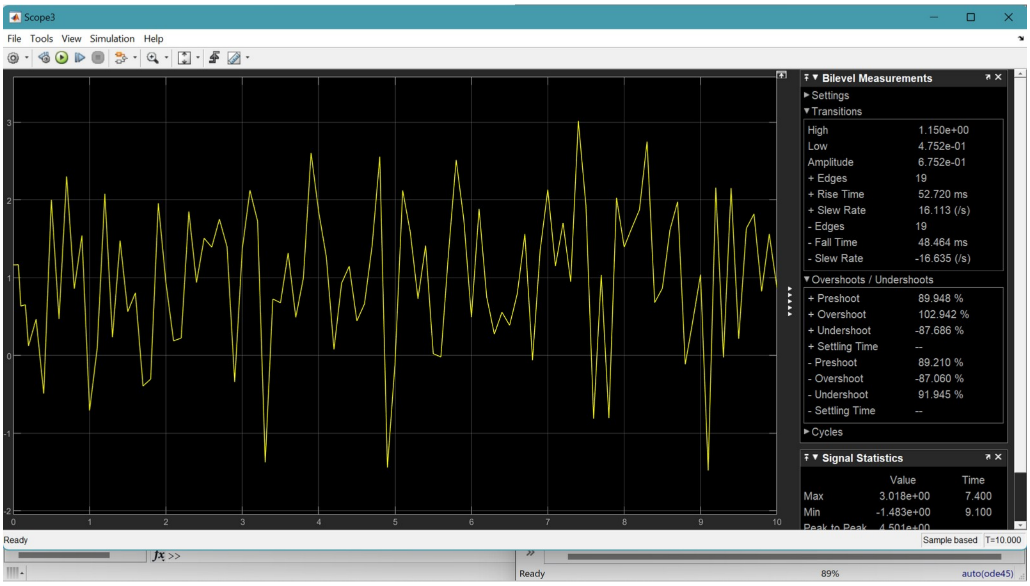

Figure 3. LQT Step Response Display with Noise

Figure 3 depicts the step response of a BSG-23 DC motor controlled by LQR with noise interference. Under these conditions, the system output experiences significant fluctuations due to noise that interferes with the input signal. Although the system achieved an amplitude of 6.75, this indicates that the system did not fully reach the setpoint. With a measurable rise time of 52.720 ms, the system experienced a high overshoot of 102.942% and a significant undershoot of -87.686%. This condition indicates that noise can drastically affect

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the stability of the system, so the LQR controller needs to be repaired to address noise issues that interfere with system performance.

3.4. Noise Free LQT Simulation Results

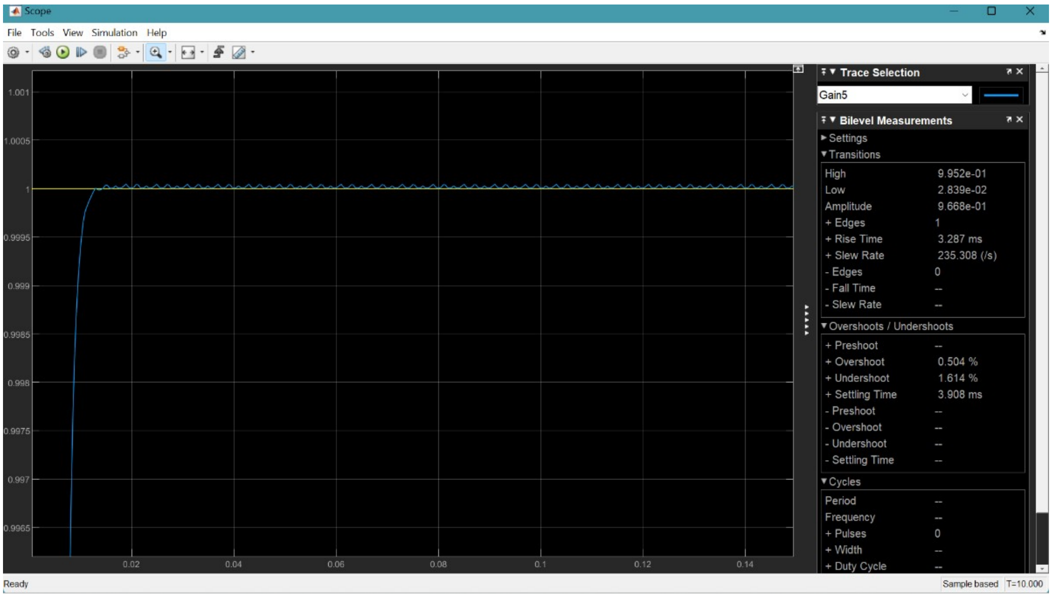

Figure 4. LQT Step Response Display Without Noise

Figure 4 shows the step response of a BSG-23 DC motor controlled using a noiseless LQT. The results show an amplitude of 9.66, which can be rounded to 1, indicating that the system achieved the setpoint very well. The rise time recorded at 3,387 ms, shows a very fast response, faster than the results of the noise-free LQR system. However, the overshoot was recorded at 0.504% and the undershoot was 1.614%, which indicates a slight decline in the performance of the LQT system despite being very good overall.

3.5. LQT Simulation Results with Noise

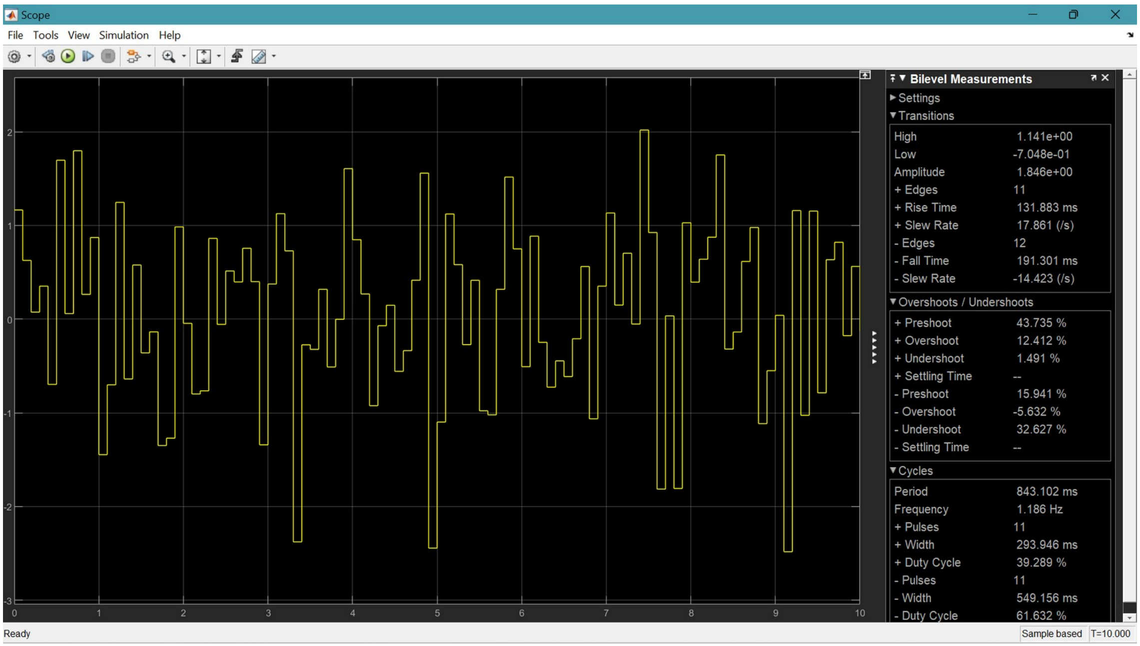

Figure 5. LQT Step Response Display with Noise

In Figure 5, it can be seen that the step response of the BSG-23 DC motor controlled with LQT under noise conditions shows considerable fluctuations. The amplitude was recorded at 1.84, which indicates that the system has not yet succeeded in reaching the setpoint. The system experienced a longer rise time of 131.883 ms and showed an overshoot of 12.412% and a sizable undershoot of 32.627%. These fluctuations indicate that the

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influence of noise on LQT systems is more significant than on LQR systems, which needs to be considered in further development to improve the system's resistance to noise interference.

Table 1. Comparison of Step Response on LQR and LQT System Models

It	System Model	Amplitude	*Rise Time*	*Overshoot*	*Undershoot*
1	Noiseless LQR	0,99	1.109 s	0,505%	0,505%
2	LQR with Noise	6,75	52.720ms	102,942%	-87,686%
3	Noiseless LQT	9,66	3.387ms	0,504%	1.614%
4	LQt with Noise	1.84	131.883ms	12.412%	32.627%

4. Conclusion

The simulation results show that the use of LQR in the BSG-23 DC motor provides more optimal performance compared to the standard DC motor model. Systems using LQR managed to achieve setpoints with stable amplitude, optimal rise time, and minimal overshoot and undershoot. In contrast, the LQT system, despite providing faster results with a shorter rise time (3,387 ms), still showed a slight increase in overshoot and undershoot that needed further improvement.

The comparison between LQR and LQT also shows that although both have advantages, LQR is more resistant to noise in the system than LQT. This suggests that for applications involving interference or noise, the use of LQRs is preferable, while LQTs are more suitable for more ideal systems without much external interference.

This research makes an important contribution to the development of DC motor control based on Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT), which can be implemented in various industrial applications that require high stability and response speed.

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