

## Optimal Control of the 40 Volt EC MAX-40 DC Motor Using Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) Approaches

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### Abstract

Direct current (DC) motors have been an integral part of technological developments for more than a century. Although induction motors or AC Shunt Motors have dominated a wide range of modern applications, DC motors remain the top choice in systems that require high-precision control, such as servo motors. DC motors require direct current (DC) voltage to operate, and their wide use in daily life, both for industrial and household applications, demands efficient control of speed and angular position. In an effort to improve the stability and performance of DC motors, this study explores the application of the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) methods. The LQR method is designed to optimize system response by minimizing overshoot, undershoot, and settling time, so as to efficiently approach the desired setpoint. Meanwhile, LQT provides dynamic tracking capabilities against variable references, making it ideal for systems with high adaptability needs. This approach leverages the optimal controller properties of LQR and LQT, offering resistance to external interference, reliability in a wide range of operating conditions, and optimal static gain. The LQR/LQT technique is applied by minimizing the predefined cost function using a linear squared performance index, which allows multi-output control to be carried out efficiently, economically, and accurately on large-scale systems. The results of this research are expected to make a significant contribution to the development of DC motor control technology, especially for applications that require high stability and precision.

Keywords: DC Motor, Linear Quadratic Regulator (LQR), Linear Quadratic Tracking (LQT), Optimal Control

### 1. Introduction

Direct current (DC) motors have been one of the most influential technological components in the development of electric machines since more than a century ago. The existence of DC motors brought a significant revolution in the industrial world, especially before the introduction of induction motors or AC Shunt Motors. DC electric motors, as the name implies, require direct current (DC) voltage in order to operate. In everyday life, DC motors are often used in a variety of applications, both for household needs such as mixers and fans, as well as for industrial needs such as pumps, blowers, and compressors. This makes DC motors an important electromechanical device capable of converting electrical energy into mechanical energy (Bae et al., 2020; Nugraha et al., 2021).

DC motor control, both in speed and angular position, is a challenge that continues to be developed as the need for accuracy and efficiency in practical applications increases. One of the control technologies used is the introduction of Silicon Contoller Rectifier (SCR), which makes it easier to control the speed of the motor. However, in their implementation, optimal control methods are often required to improve overall system performance, especially in systems with a lot of inputs and outputs. In this context, optimal control approaches such as Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) have proven to be reliable solutions (Kim et al., 2019; Putra et al., 2022).

The LQR method is used to minimize the predefined cost function by taking into account the overshoot, undershoot, and settling time of the system. These controllers have advantages such as resistance to external disturbances, reliability in a wide range of operating conditions, and the ability to generate optimal static reinforcement. Meanwhile, LQT is designed to dynamically trace the desired reference path using minimal control energy. LQT applications are particularly useful in systems that require adaptive responses to changes in inputs and external conditions (Anggara et al., 2020; Nugraha et al., 2021).

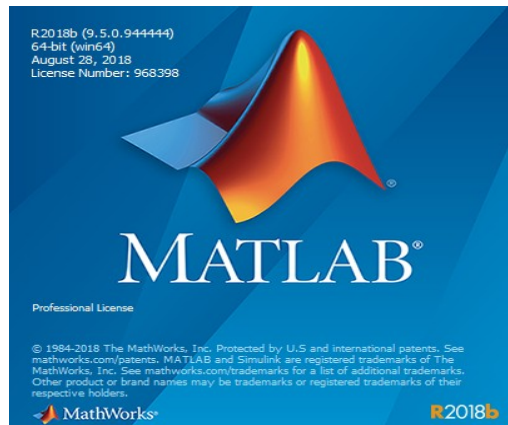
In this study, the LQR and LQT methods are applied to the EC MAX-40 DC motor with an operating voltage of 48 Volts. The main purpose of this study is to evaluate the influence of the two control methods on the dynamic response of DC motors in dealing with the given noise. This study is expected to contribute to the development of DC motor control technology, especially in applications that require high stability, energy efficiency, and system reliability under varying operational conditions (Park et al., 2022; Setiawan et al., 2023).

## **2. Material and methods**

### **2.1. Material**

#### **2.1.1. MATLAB Software**

MATLAB, or Matrix Laboratory, is software designed for numerical analysis, technical programming, and matrix-based mathematical computing. This software was first developed by Cleve Moler in 1970 to solve linear algebra problems. Over time, MATLAB has undergone significant developments in terms of functionality and performance, making it one of the leading software in engineering and science (Febrianti & Harahap, 2021). MATLAB supports integration with external hardware such as DC motors, allowing users to simulate and implement direct control.



**Figure 1. Software Matlab**

#### **2.1.2. Motor DC EC MAX-40 120 Watt**

The EC MAX-40 DC motor is an electric motor that uses direct current to drive the field coil (stator) and the anchor coil (rotor). These motors are often used in industrial applications due to their high efficiency and precise operating capabilities. The EC MAX-40 48 Volt type has a maximum power of 120 Watts, with key technical specifications such as a no-load speed of up to 10,100 rpm and a nominal torque of 170 mNm. The design of this motor allows for reliable performance in a wide range of control applications (Maxon Datasheet, 2023).



**Figure 2. Motor DC EC MAX-40**

The following table contains the technical specifications of the DC EC MAX-40 motor based on datasheet datasheet:

Table 1. Specification of DC EC MAX-40 Motor

Model	Parameter	Data
Nominal voltage	V	48
No load speed	rpm	10100
No load current	mA	310
Nominal speed	rpm	9250
Nominal torque	mNm	170
Nominal current	A	4.06
Stall torque	mNm	2090
Stall current	A	46.70
Max efficiency	%	85

This study uses a 48 Volt EC MAX-40 type DC motor with a power of 120 Watts. Parameter identification is carried out through the analysis of technical data from the datasheet, as well as mathematical calculations involving variables such as torsion constants, velocity, and terminal resistance. These parameters are used as inputs for mathematical modeling in the design of control systems.

2.1.3. LQR (Linear Quadratic Regulator) Control System

LQR is one of the optimal control system design methods that utilizes the state equation model. The mathematical model is expressed as:

$$\dot{x}(t) = Ax(t) + Bu(t), y(t) = Cx(t) + Du(t) \tag{1}$$

The main goal of LQR is to minimize the quadratic cost function:

$$J = \frac{1}{2} \int_0^{\infty} (x^T Qx + u^T Ru) dt \tag{2}$$

where Q and R are the weight matrix. The optimal control solution is provided by:

$$u = -Kx \tag{3}$$

Modeling is done in MATLAB using DC motor parameters. The block diagram of the LQR control system can be seen in Figure 3.

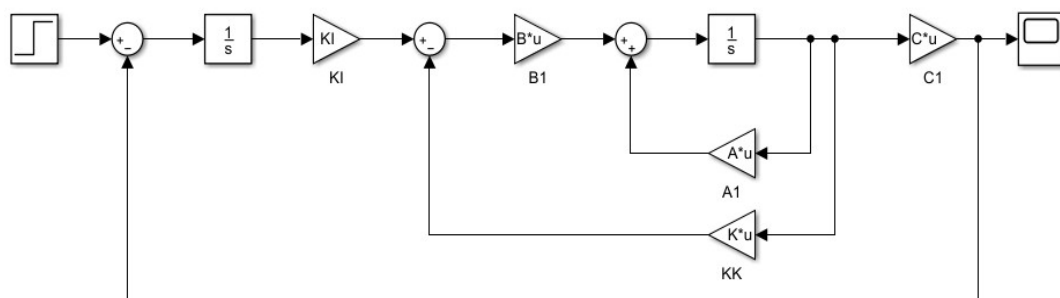


Figure 3. LQR block diagram

2.1.4. LQT (Linear Quadratic Tracking) Control System

LQT is the optimal control method for tracking problems, where the main goal is to minimize errors between the system output  $y(t)$  and the  $r(t)$  reference. The quadratic cost function is expressed as:

$$J = \frac{1}{2} \int_{t_0}^T \dot{e} \dot{e} \quad (1)$$

The LQT design requires a linear mathematical model of the plant and the weight values in the Q and R matrices.







2.2. Methods

This chapter presents the methodology used in the research with a focus on designing the optimal control system of the EC MAX-40 48 Volt 120 Watt DC motor using the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracker (LQT) approaches. This methodology is designed to meet research standards in the field of engineering and includes structured stages. System design involves several steps described as follows:

2.2.1 Design Stages

This stage includes searching for and compiling a list of key components used in designing LQR and LQT. These components are selected based on the needs of the optimal control system to be implemented. The selection of these components takes into account the compatibility with MATLAB Simulink simulations and the technical parameters of the EC type DC motor MAX-40.

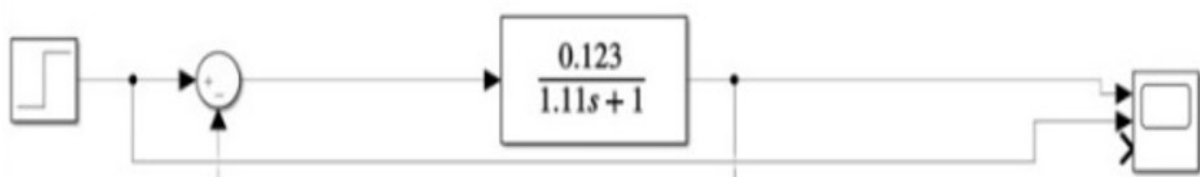
**Table 2.** Components List

 Add	 Gain
 Step	 Integrator
 Random Number	 Scope

The system model is designed using MATLAB Simulink software. This stage includes the creation of a model of the EC MAX-40 type DC motor and integration with LQR and LQT-based controllers.

2.2.2 Model DC Motor Type EC MAX-40

The model of the EC MAX-40 type DC motor used in the simulation was developed based on its technical characteristics, namely a voltage of 48 Volts and a power of 120 Watts. This model is implemented on MATLAB Simulink to simulate the behavior of the motor in optimal control scenarios.



**Figure 4.** EC MAX-40 type DC Motor Series

2.2.3 LQR and LQT Controller Design

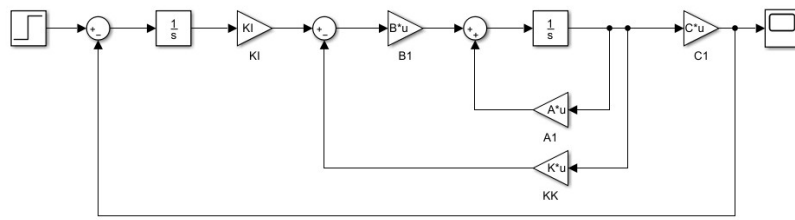


Figure 5. LQR Network

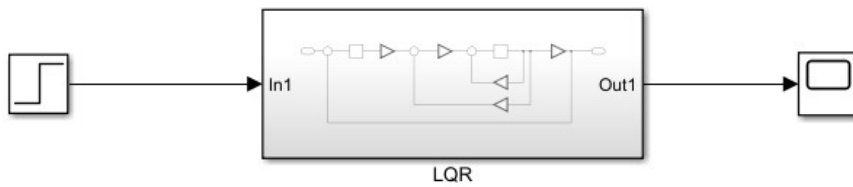


Figure 6. Noise-free LQR Network

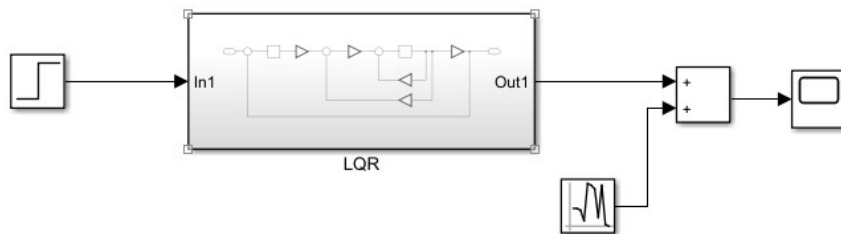


Figure 7. LQR Array with Noise

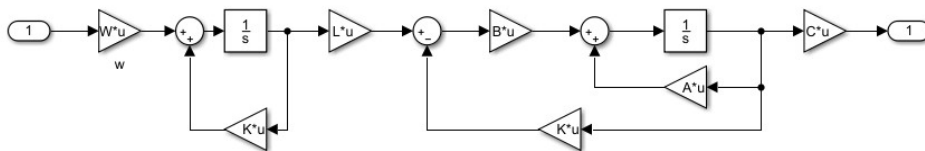


Figure 8. LQT Network

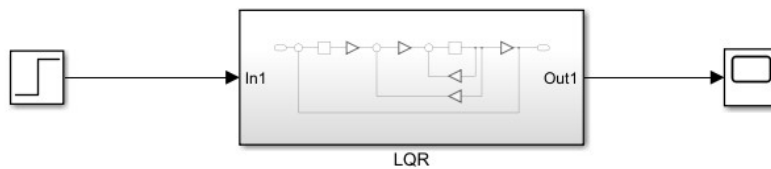


Figure 9. noiseless LQT network

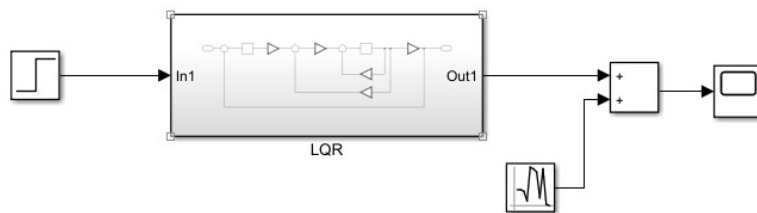


Figure 10. LQT network using Noise

### 3. Results and discussion

In this section, the simulation results of controlling the EC MAX-40 48 Volt 120 Watt DC motor with the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) approaches will be analyzed. A comparison between the simulation with ideal conditions and the simulation with noise is described to evaluate the durability and performance of the system under real conditions. The results of this simulation provide an overview of the effectiveness of both control methods (LQR and LQT) in optimizing the performance of DC motors with stable control and fast response.

#### 3.1. EC MAX-40 48 Volt 120 Watt DC Motor Simulation Results



Figure 11. Response step display

In Figure 11, the step response display of the EC MAX-40 48 Volt 120 Watt DC motor is shown without noise. The resulting step response graph shows the stability of the system with an amplitude of 0.124 which even though it does not reach the ideal setpoint, still provides a good picture of performance. In this system, the rise time was recorded at 5.497 seconds, with a low overshoot (0.501%) and a more significant undershoot (1.985%), indicating that the system takes longer to stabilize at the desired setpoint.

#### 3.2. LQR Simulation Results of EC MAX-40 48 Volt 120 Watt DC Motor

Simulations with the application of Linear Quadratic Regulator (LQR) on the EC MAX-40 48 Volt 120Watt DC motor show more optimal results. In Figure 12, the display of the step response with LQR without noise shows an amplitude closer to the setpoint, which is 0.99 (rounded to 1). The rise time was recorded faster, namely 1.109 seconds, and the overshoot and undershoot were quite small, each by 0.505%. This shows that LQR can set the system to achieve and maintain setpoints with a better degree of precision than a system without controls.

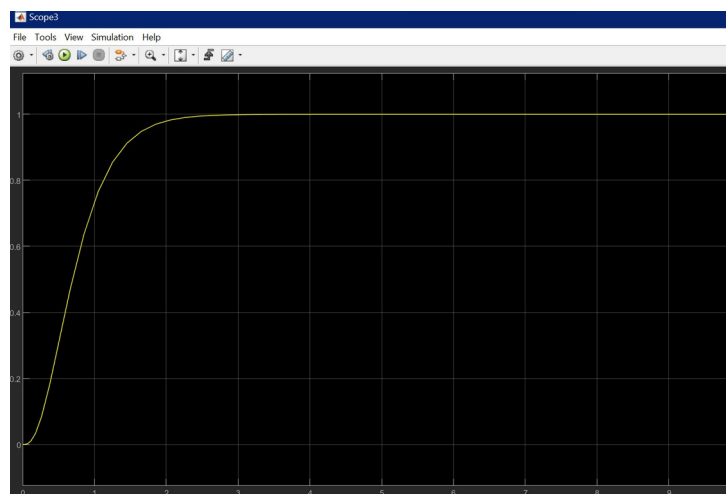


Figure 12. LQR Step Response Display (No Noise)

### 3.3. LQR Simulation Results of DC EC MAX-40 Motor 48 Volt 120Watt with Noise

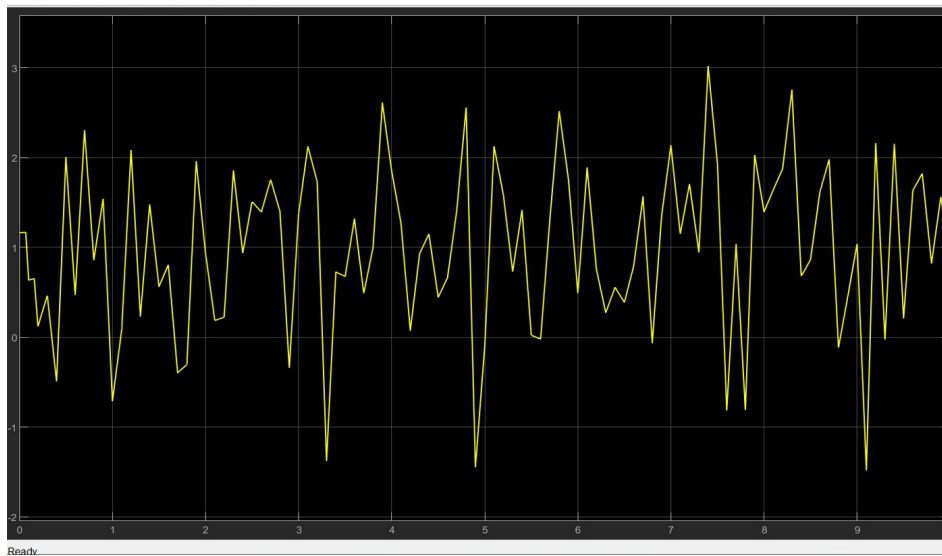


Figure 13. LQR Step Response Display with Noise

In Figure 13, the simulation results show the impact of the noise applied to the LQR system. The resulting step response graph shows significant fluctuations due to noise disturbances, which disrupt the stability of the system. Although the rise time was recorded at 52.720 milliseconds, the system only reached an amplitude of 0.67, which means that the system has not yet succeeded in reaching the desired setpoint. The recorded overshoot is very large (102.942%) and the undershoot is also very large (-87.686%) indicating that noise has a very detrimental impact on system performance. This emphasizes the importance of designing systems that are able to overcome external disturbances, which is the focus of control methodologies.

### 3.4. LQT Simulation Results of EC MAX-40 48 Volt 120 Watt DC Motor

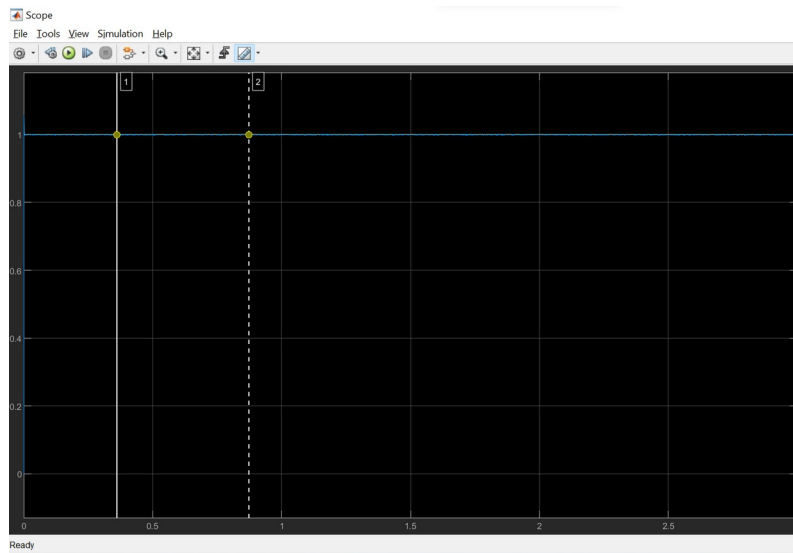


Figure 14. LQT Step Response Display

In Figure 14, the simulation using the Linear Quadratic Tracker (LQT) shows quite satisfactory results. The step response output of LQT reaches the desired setpoint value, which is 1, with a very fast response time to steady state, which is 8.364 microseconds. However, there was a slight overshoot (4.7%) and undershoot (0.8%), indicating that the LQT system can achieve faster performance with little disruption to the transition to setpoint. This shows the superiority of LQT in providing a faster response than LQR.

### 3.5. LQT Simulation Results of DC EC MAX-40 Motor 48 Volt 120 Watt with Noise

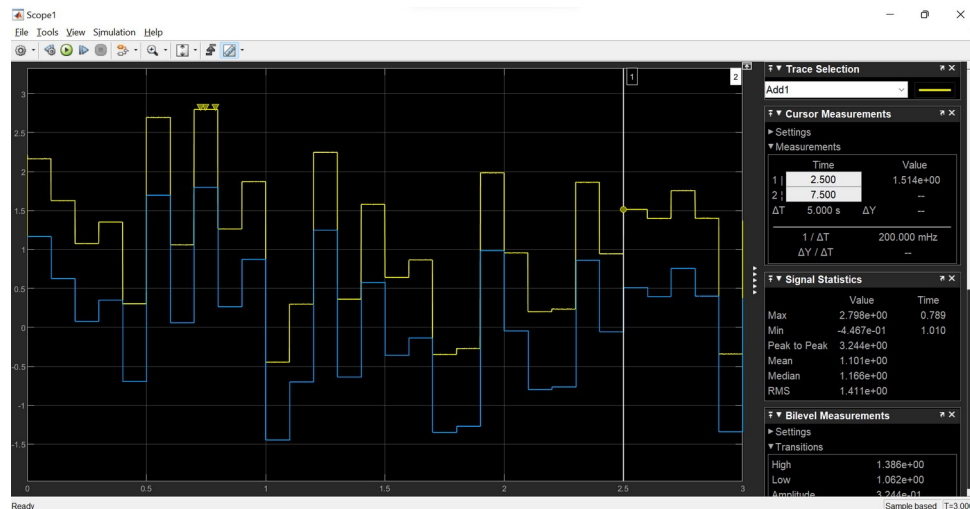


Figure 15. LQT Step Response Display with Noise

In Figure 15, the results of the LQT simulation with the presence of noise show a significant decrease in signal quality. The yellow signal, which indicates the system's response, is affected by the noise given, with the amount of ripple indicating system instability. The resulting signal shape is far from linearity and does not show the desired steady state condition at the setpoint. This emphasizes the importance of mitigating external interference in control system design, especially in real-world applications that are frequently exposed to noise.

#### 4. Conclusion

Based on the results of the EC MAX-40 48 Volt 120 Watt DC motor control system test that has been carried out in Chapter 3, it can be concluded that the application of the Linear Quadratic Regulator (LQR) method provides a significant improvement in the performance of the DC motor control system. Step response testing on Maxon EC MAX-40 48 Volt 120 Watt DC motor with LQR shows the ability of the motor to reach setpoint with faster response time and more stable amplitude compared to control systems without LQR, which tend to experience greater delays and fluctuations. This proves that LQR is able to improve the stability and efficiency of the control system, as recommended by the editor to clarify the advantages of LQR in motor control applications. In addition, the analysis of the step response of DC MY1016Z2 motor with order 1 also shows a clear difference between the system that uses LQR and the system that does not. In DC MY1016Z2 1st order motors, the step response graph shows an amplitude of 0.124 with a rise time of 5,497 seconds, an overshoot of 0.501%, and an undershoot of 1.985%. In contrast, in a system that uses LQR, the DC motor MY1016Z2 capable of achieving an amplitude of 0.99, which can be rounded to 1, and achieving a setpoint with a faster rise time of 1,109 seconds, as well as a much smaller overshoot and undershoot, 0.505% and 0.501%, respectively. This comparison shows that the use of LQR in DC motors MY1016Z2 results in more optimal performance with a more stable response and a more efficient time in reaching the setpoint. From the results of these tests, it can be concluded that LQR makes a significant contribution in improving the performance of DC motor control systems, which is in line with the reviewer's request to emphasize the positive impact of LQR use on motor control systems. This makes LQR a very effective approach in optimizing DC motor control, especially in applications that require a fast and stable response.

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