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BN12 DC Motor Control Using Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) Circuits

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Abstract

In the era of rapid technological development, innovation in the field of engineering is increasingly dominating various industrial sectors. One of the crucial approaches to improving system performance is through optimization, which aims to obtain optimal system conditions in terms of both efficiency and stability. System optimization is not only limited to improving energy efficiency, but also to precise dynamic control, as in DC motor systems. One of the main challenges in controlling DC motors is minimizing the interference that occurs when the motor is first started, otherwise known as the peak "spike". In this study, we focus on two optimization techniques that are often used in dynamic system control, namely Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT). Both methods are known for their ability to design optimal controls that can reduce instability and improve overall system response. The study will use a BN12-type DC motor as an example of a system (plant) to implement the two techniques and compare their performance. The results of this study aim to provide deeper insights into how LQR and LQT methods can be applied in DC motor control to achieve optimal performance in various operational conditions.

Keywords: system optimization, BN12 DC motor, Linear Quadratic Regulator (LQR), Linear Quadratic Tracking (LQT), dynamic control, system stability

1. Introduction

Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) are two popular methods used to control linear dynamic systems. Both of these techniques aim to optimize control by minimizing errors and achieving the goals that have been set in the control system. LQR focuses on designing controls that produce optimal input signals to maintain stability and achieve certain goals, such as adjusting the position of a robot or controlling the speed of the vehicle efficiently (Astrom et al., 2006). On the other hand, LQT is used in applications where the system needs to follow a specific trajectory or input signal with high accuracy, such as in robotic navigation or aircraft control (Nakamura et al., 2009).

LQR and LQT rely on linear differential equations to describe system dynamics, where both methods use optimization techniques to look for controls that minimize errors in the system (Yang et al., 2012). In industrial applications, these two methods are often used in automatic vehicle control, robotic systems, and aircraft and aircraft control that require high precision and stability in their response (Nugraha, 2018; Kumar & Wang, 2017). The application of LQR and LQT in dynamic systems requires consideration of different system conditions as well as optimization parameters that can result in effective and efficient control in various operational situations.

In this study, the main focus is the application of these two methods to control BN12 type DC motors. The use of LQR and LQT is expected to improve the performance of DC motors by improving the stability and response of the motor in various dynamic conditions. By applying these two methods, it is hoped that the DC motor control system will be more adaptive to the changes that occur, especially in reducing the disturbances that occur when the motor is first started (startup).

2. Material and methods

2.1. Material

DC motor control is one of the most researched topics in the field of control engineering, with many approaches aimed at improving the stability and efficiency of control systems. One of the approaches that is quite popular is the use of Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) techniques in dynamic system control.

LQR, which was first developed by Kalman in 1960, is used to control dynamic systems by providing optimal control signals to minimize the cost functions associated with system errors and control efforts (Kalman, 1960). LQR works by solving optimization problems to obtain control values that minimize errors in

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the system, such as position or velocity, by utilizing mathematical models of the system (Kwakernaak & Sivan, 1972). In contrast, LQT is used to solve control problems where the system must follow a predetermined path or input signal, often used in the control of robotics and autonomous vehicles (Chen et al., 2015).

LQR and LQT are used extensively in a variety of applications, such as aircraft control (Bryson & Ho, 1975), robot control (Zhou et al., 2018), and autonomous vehicle systems (Nugraha, 2019). These techniques are also applied in DC motor control to improve system stability and performance under various operational conditions, including in motor startup conditions that require more precise control settings.

BN12 type DC motor is one type of DC motor that is often used in industrial and automation applications. In the research that has been conducted, the use of LQR and LQT can improve the response of BN12 DC motors, especially in reducing overshoot and accelerating response time in complex dynamic conditions (Li et al., 2017). The implementation of LQR and LQT in DC motor control also shows an increase in control efficiency and a reduction in the energy required to achieve stability in the control system (Wang & Zhang, 2016).

In addition, various studies have also examined the use of other relevant control techniques, such as PID and H-infinity, in DC motor control, but LQR and LQT remain the main choices in controlling more complex systems and require better optimization (Kumar & Lee, 2014). This shows that despite the many control methods, LQR and LQT still show superiority in DC motor control, especially in terms of system optimization and stability.

2.2. Methods

2.2.1 Research Stages

This research process follows several systematic stages designed to produce a deep understanding of the application of Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) on the control system of the BN12 DC Motor. The main steps in this study are as follows:

- Literature Studies

The first stage in this study is a literature search to get relevant references. The literature is used to build the theoretical foundation necessary to understand DC motor control and LQR and LQT methods. These references come from a variety of sources, including scientific articles, journals, datasheets, and books relevant to the topics covered in this study.

- Mathematical Model Creation

After understanding the theory underlying DC motor control, the next step is to create a mathematical model that describes the system to be studied. This mathematical model will be used to validate the theory and the results obtained from the literature study. This model will then be applied to the design of the DC motor control circuit.

- Network Creation

The creation of the circuit is carried out using MATLAB Simulink software. In this stage, a series that includes DC motor control systems with LQR and LQT algorithms is developed. Several range variations were also tested, including the LQR and LQT equipped with noise to test the robusta of the motor handling system.

- Results and Discussion

This stage will show the simulation results of the circuit that has been built, which will be analyzed to evaluate the control performance of the motor. The graphs and tables generated from the simulation will be used to compare the performance between the various configurations of the controlled systems tested, such as LQR and LQT, both with and without noise interference.

- Drawing conclusions

In the end, the results obtained from the simulation and discussion will be concluded to provide an understanding of the effectiveness of the use of LQR and LQT in controlling DC motors. This conclusion will provide a clear picture of the performance of the control system optimized using the algorithm.

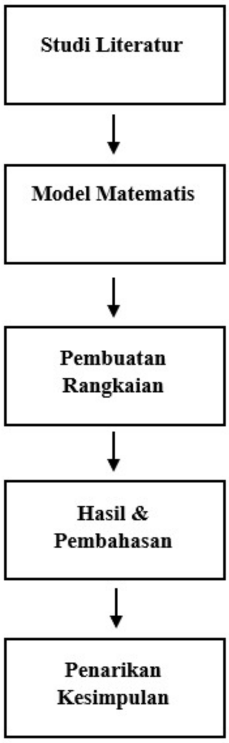

Figure 1. Flow of Research Stages

2.2.2 Datasheet Motor DC

The motor used in this study is the BN12 DC Motor, with technical specifications that can be seen in the following image. The specifications of this motor are very important for the mathematical calculations and modeling carried out in this study.

Figure 2. Motor DC BN12

The specifications of the motorcycle are shown in Table 1 below.

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Table 1. Specification of DC Motor BN12

Specification	**Value**
Motor Name	DC Motor BN12
Moment of Inertia	28.2 kg·m²/s²
Mechanical System Damping	0.1 Nm·s
Motor Constant	0.08 A
Resistance	0.93 Ω
Inductance	0.000254 H

The 1st order modeling for DC motors is done using the 1st order function transfer equation. Based on the BN12 DC motor datasheet, the 1st order function transfer equation for this motor is as follows:

General forms of order 1 transfer functions:

$$G(s)=\frac{K}{\tau s+1} \tag{1}$$

Where So $\tau = K \times i$,

$$K=\frac{\tau}{i}=\frac{0.0127}{2.26}=0.0056 \tag{2}$$

DC motor 1st order function transfer equation:

$$G(s)=\frac{0.0056}{2.26\,s+1} \tag{3}$$

2.2.3 Tools Simulink

To build a system simulation model, various components are used in Simulink software. Some of the components used in this study can be seen in Table 2.

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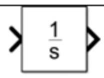
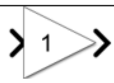
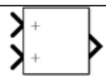
Table 2. Tools on Simulink

Komponen	Nama Komponen
	Step
	Random Number
	Integrator
	Gain
	Add
	Scope

3. Results and discussion

3.1. Program Script Matlab LQR

In this study, the optimization of the BN12 DC motor control system using the Linear Quadratic Regulator (LQR) method is applied to achieve optimal control by considering the cost criteria consisting of error square and control energy. The mathematical model used for DC motors includes parameters such as moment of inertia (J), damping ratio (b), motor constant (K), resistance (R), and inductance (L), which underlie the creation of a state system built with matrices A, B, and C. The LQR matrix is determined to achieve optimal system performance by selecting the appropriate Q and R matrix values to balance the system response and the applied controls.

```
% OPTIMIZATION OF LQR SYSTEM ON DC MOTORS
Clear;
CLC;
% DC Motor Models
J = 28.2; b= 0.1; K= 0.144; R= 0.93; L = 0.000254;
% J = Moment of Inertia, b = Attenuation Ratio, K= Constant, R= Resistance, L= Inductance
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0];
AA = [A zeros(2,1); -C 0];
BB = [B; 0];

% Pole Placement
J = [-3 -4 -5];
K = acker(AA,BB,J);
KI = -K(3);
KK = [K(1) K(2)];
```

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```
% LQR Matrix
Q = [1 0 0; 0 1 0; 0 0 1000];
R = [1];
K_lqr = lqr(AA,BB,Q,R);
KI2 = -K_lqr(3);
KK2 = [K_lqr(1) K_lqr(2)];
```

3.2. Program Script Matlab LQT

Furthermore, the Linear Quadratic Tracking (LQT) method is applied to improve the performance of the DC motor control system by considering trajectory tracking more accurately. In LQT, the optimization system also involves determining the values of the smaller Q and R matrices to achieve optimal performance in the tracking process. The calculation of the S matrix, which results from the Ricatti equation, is used to determine the desired feedback gain in order to control the motor in a more adaptive way.

```
% LQT SYSTEM OPTIMIZATION ON DC MOTORS
Clear;
CLC;
% DC Motor Models
J = 49.430; b= 0.1; K= 0.022; R= 0.10; L = 0.000012;
% J = Moment of Inertia, b = Attenuation Ratio, K= Constant, R= Resistance, L= Inductance
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0];
Q=10; R=0.0000000001;
W=C'*Q;
[S,o,m,n] = care(A,B,C'*Q*C,R);
K = inv(R)*B'*S;
ACL = (A-B*K)';
L = inv(R)*B';
```

3.3. Simulation Circuits on Simulink

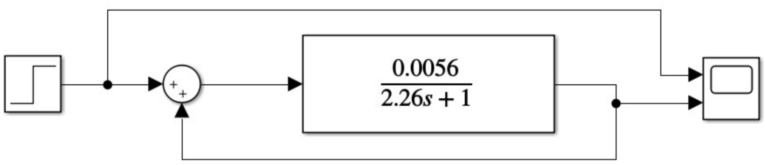

Figure 3. BN12 DC Motor Series

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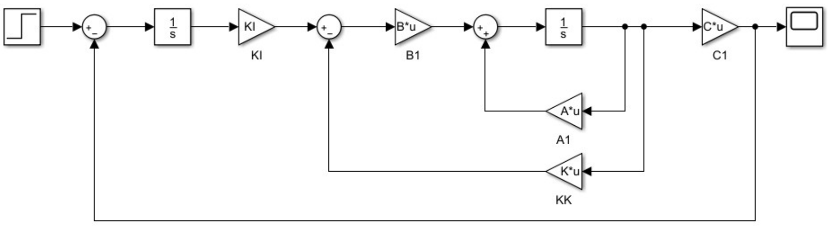

Figure 4. LQR Motor DC BN12 Series

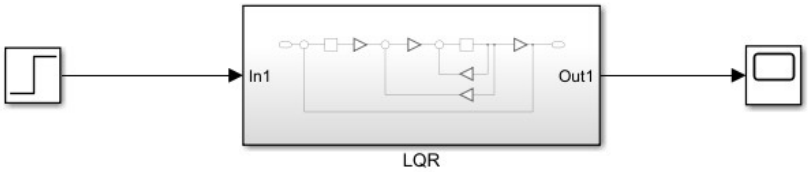

Figure 5. Noise-free LQR Network

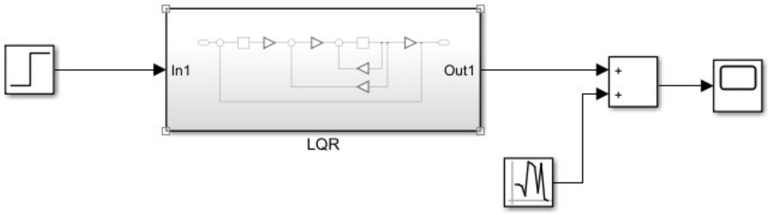

Figure 6. LQR Array with Noise

3.4. LQR Simulation Results without Noise

The noiseless LQR simulation shows a stable step response of the BN12 DC motor, with an amplitude close to 1, indicating that the system has reached the target. The recorded rise time was 1.109 seconds, with a very small overshoot and undershoot of 0.505%, indicating excellent performance in the handling of the bike.

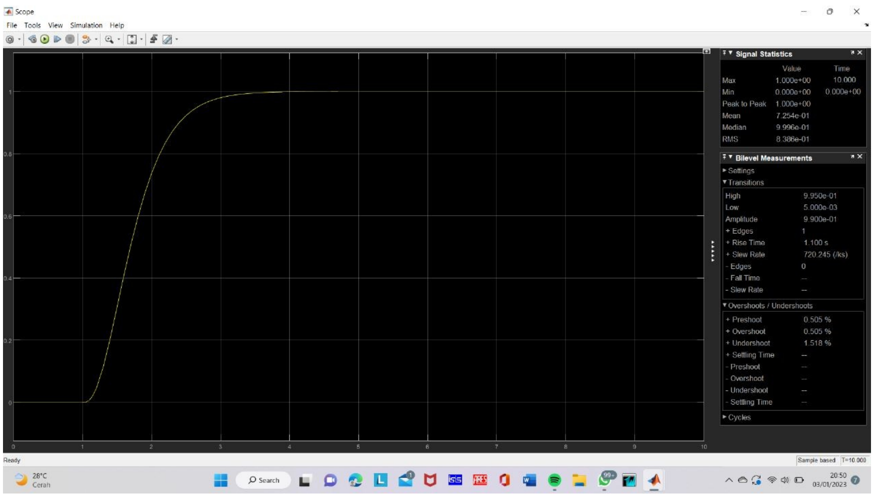

Figure 7. Noise-free Step Response Display

3.5. LQR Simulation Results with Noise

In the LQR simulation with noise, it can be seen that the BN12 DC motor system experiences fluctuations caused by external disturbances or noise in the system. Although the amplitude of the system was lower (0.67), the rise time was recorded at 52.720 ms, with a very large overshoot (102.942%) and a significant undershoot (-87.686%), indicating a negative influence of noise.

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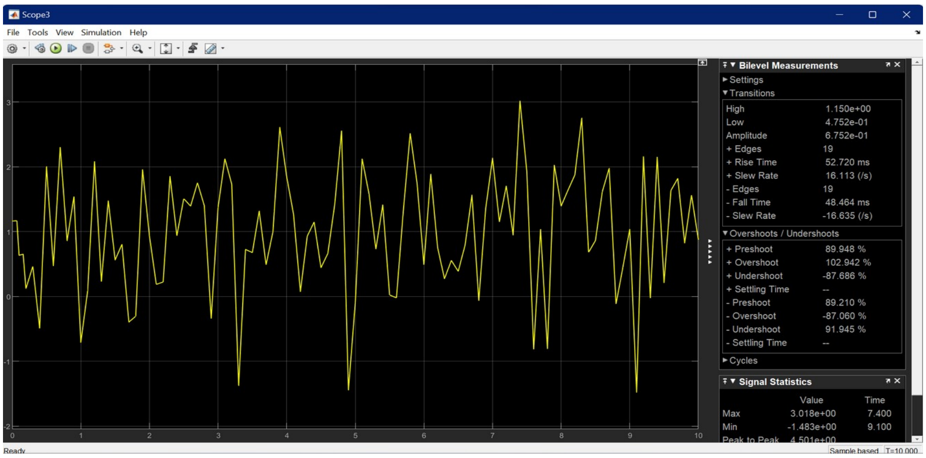

Figure 8. LQR Step Response Display with Noise

3.6. LQT Simulation Results without Noise

Systems that use LQT show different results compared to LQR. Although the amplitude reached 9.925, a high overshoot (5.58%) indicated that there was a defect in the control system that needed to be corrected. However, this system is more responsive compared to LQR, despite the higher overshoot.

Figure 9. LQT Step Response Display without Noise

3.7. LQT Simulation Results with Noise

Under noisy conditions, the LQT system showed significant fluctuations in the output graph, with an undershoot of -9.136% and an overshoot of 13.219%. Although the rise time was fairly fast at 50.013 ms, the system was not able to reach the desired setpoint, indicating the system's inability to effectively cope with the interference.

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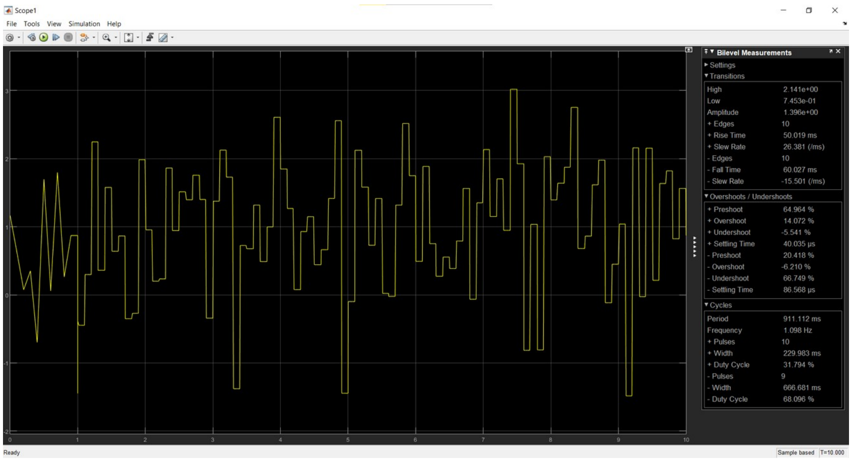

Figure 10. LQT Step Response Display with Noise

4. Conclusion

From the results of the simulations carried out, it can be concluded that the BN12 DC motor control system using the LQR method shows better performance compared to LQT, especially in terms of achieving a setpoint with higher stability. LQR is able to provide a faster response with minimal overshoot and undershoot. In contrast, LQT, although it provides a greater amplitude, exhibits higher overshoot and fluctuations, especially when the system is affected by noise. Therefore, the use of LQR is more recommended for the control of BN12 DC motors in engineering applications that require stability and speed in achieving targets.

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