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Mathematical modeling and simulation of open loop and closed loop systems for a second-order Rotary type S-50-39 motor: Ziegler Nichols

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ABSTRACT

This study examines optimal control methods for a servo motor, focusing on performance evaluation in both open-loop and closed-loop conditions. In open-loop configuration, the PID method is identified as the best solution due to its ability to provide superior system stability and response. For closed-loop systems, a proportional (P) control method is selected based on its fast rise time and very small overshoot, making it suitable for applications requiring rapid response and high accuracy. The parameter values for the closed-loop system are determined using the Routh-Hurwitz criteria to ensure system stability. The findings of this research provide practical guidance for the efficient and stable implementation of DC motor control.

Key Word: *Modeling 1, simulation 2, PID 3, Ziegler Nichols Method 4, Routh Hurwitz 5, Servo motor 6.*

I.INTRODUCTION

In modern industrial applications, precise control of motor systems is essential for achieving high performance and reliability. One of the widely used motor types in various sectors is the Rotary Type S-50-39 motor. This motor is known for its robust performance and adaptability in a range of environments. Understanding the dynamic behavior of this motor through mathematical modeling and simulation is crucial for optimizing its performance in both open loop and closed loop control systems.

Mathematical modeling provides a framework for describing the dynamic characteristics of the Rotary Type S-50-39 motor. By representing the motor as a second-order system, we can capture its response to various inputs and disturbances. This modeling is foundational for designing control systems that ensure the motor operates efficiently and accurately.

Open loop control systems, while simpler to implement, do not account for feedback, making them less effective in compensating for disturbances or variations in system parameters. On the other hand, closed loop control systems incorporate feedback mechanisms to continuously adjust the motor's input, thereby maintaining the desired output despite external perturbations. The comparative analysis of open loop and closed loop systems provides valuable insights into the advantages and limitations of each approach.

In this study, we aim to develop a comprehensive mathematical model of the Rotary Type S-50-39 motor and simulate its performance under open loop and closed loop configurations. By analyzing the transient and steady-state responses, we seek to identify optimal control strategies that enhance the motor's operational efficiency and precision. The findings from this research will contribute to the development of more effective control systems for rotary motors in various industrial applicationss.

II.METHODOLOGY

Several related research papers are being reviewed and studied to improve the control system model of the existing motor to achieve the best efficiency point. The results of each project will be summarized, and all advantages and limitations will be compared.

1. **Mathematical model of servo motor**

In this work, the schematic of the S-50-39 servo motor is analyzed. From this schematic, a mathematical model of the motor will be derived to determine the modeling of this motor. The obtained mathematical model will be used to determine the necessary datasheet variables to subsequently obtain the transfer function.

2. **Obtaining the transfer function from the datasheet**

In this step, inputting variable values and obtaining the transfer function using equation

3. **Modeling a motor in open loop circuit**

The mathematical model forms the basis for creating a control system model. From here, it is necessary to first assemble the system. This involves creating a basic circuit with a reference scheme as the input, the transfer function as the plant, and a scope/display as the output.

4. **Modeling a motor in close loop circuit**

In this closed-loop circuit, what differentiates it from the open-loop circuit is the presence of a feedback plan. Mathematically, it can be said that the output of the transfer function is a real number, whereas the feedback is an imaginary number. Values need to be added, from which the rise time value will emerge. In the case of a rotary motor, the feedback value can be obtained by multiplying the transfer function by ϴ(s). In the transfer function that I have obtained with units in rad/s, this transfer can be multiplied by 1/s to convert it to radians.

III.RESULT & DISCUSION

Mathematical model of servo motor

In this work, the schematic of the S-50-39 servo motor is analyzed. From this schematic, a mathematical model of the motor will be derived to determine the modeling of this motor. The obtained mathematical model will be used to determine the necessary datasheet variables to subsequently obtain the transfer function.

Table 1 *Datasheet*

Variabel	Unit	Value
Resistance	Ω	6.6
Inductance	H	1.5
Frameless Rotor Inertia	Kg.m^2	1.10^-5
Torque Constant	N.m/A	0.09
BEMF Constans	V/rad.s^-1	0.0984
Damping coefficient	N.m/rad.s^-1	0.002

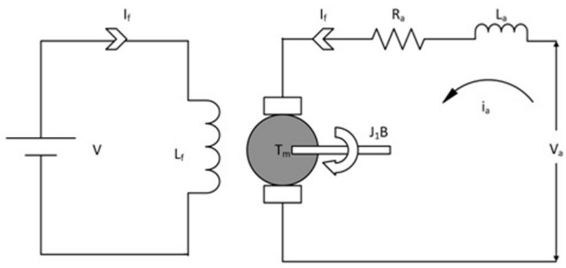

Fig. 1 *Schematic servo motor*

The voltage equation for a DC motor is;

$$\boldsymbol{V(t) = L\,\frac{di(t)}{dt} + Ri(t) + e(t)} \quad (1)$$

In the context of a servo motor, $V(t)$ represents the input voltage, *L* denotes the inductance of the motor windings, *R* signifies the resistance of the motor windings, and *i(t)* is the current flowing through the motor windings and $Kb(t)$ is the back EMF constant.

The equation of motion for the mechanical part of the motor is:

$$J\frac{d\omega(t)}{dt} + B\omega(t) = Tm(t) - TL(t)$$
(2)

When in the context of a DC motor control system, J represents the moment of inertia of the rotor, which is a measure of the rotor's resistance to changes in its rotational speed. The coefficient of viscous friction, denoted as B, quantifies the resistive force due to friction that opposes the rotor's motion. The angular velocity of the rotor is represented by $\omega(t)$, indicating how fast the rotor is spinning at a given time. The torque produced by the motor is denoted as $Tm(t)$, which drives the rotor's motion. Additionally, $TL(t)$ represents the load torque, which is the external torque applied to the motor, often from a load or external resistance the motor is working against.

The motor torque $Tm(t)$ is related to the motor current $Kt\, i(t)$ by the equation: where Kt is the motor torque constant.

Transfer function untuk kecepatan sudut Ω(s) terhadap tegangan input V(s) adalah:

$$\frac{\Omega(s)}{V(s)} = \frac{Kt}{(Js+B)(R+sL)+KtKe}$$
(3)

Laplace transform of an electrical differential equation:

$$V(s) = Ls \times I(s) + R \times I(s) + Kb \times \Omega(s)$$
(4)

Laplace transform of a mechanical differential equation:

$$Js \times \Omega(s) + B \times \Omega(s) = Kt \times I(s)$$
(5)

A servo motor is controlled using feedback to achieve the desired position or speed. A commonly used control model is the PID (Proportional-Integral-Derivative) control:

$$u(t) = Kpe(t) + Ki\int e(t)dt + Kd\frac{de(t)}{dt}$$
(6)

Where in this context, $u(t)$ represents the control signal applied to the system, while $e(t)$ denotes the error, which is the disparity between the desired and actual positions or velocities. The terms $Kp, Ki, and\ Kd$ stand for the proportional, integral, and derivative gains of the controller, respectively. These gains play crucial roles in shaping the response of the control system: Kp adjusts the immediate response to the current error, Ki addresses accumulated past errors over time, and Kd influences the system's response to the rate of change of the error. Together, these parameters contribute to the overall performance and stability of the control system, ensuring that the system behaves as intended in various operating conditions.

Obtaining the transfer function from the datasheet

In this step, inputting variable values and obtaining the transfer function using equation (3).

$$\frac{\Omega(s)}{V(s)} = \frac{Kt}{(Js+B)(R+sL)+KtKe}$$

$$Gs = \frac{0.09}{(0.00001 + 0.002)(6.6 + 1.5) + 0.09.0.0984}$$

$$Gs = \frac{0.09}{1.5 \times 10^{-5}\, S^2 + 0.003066\, S + 0.02206}$$

From the obtained transfer function, using the syntax step(Gs) will display the graph.

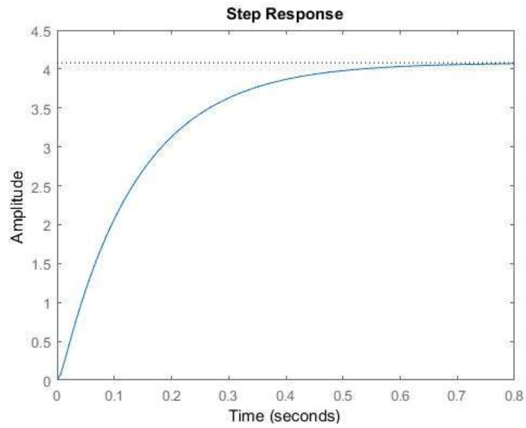

Fig. 2 *Transfer Function step response*

Modeling a motor in open loop circuit

The mathematical model forms the basis for creating a control system model. From here, it is necessary to first assemble the system. This involves creating a basic circuit with a reference scheme as the input, the transfer function as the plant, and a scope/display as the output.

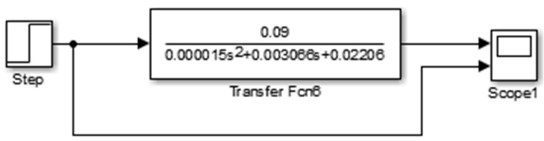

Fig. 3 *Open loop Control System*

The reference signal will be sent to the output through the transfer function as the plant and plotted according to the transfer function values.

Fig. 4 *Scope Open Loop Control System*

The graph above represents a direct plot of the values generated by the transfer function, yielding a value of 4.08. Setelah mengidentifikasi koefisien penyebut $\boldsymbol{\alpha 2}$ = 1.5×10−5, $\boldsymbol{\alpha 1}$ = 0.003066, dan $\boldsymbol{\alpha 2}$ = 0.02206, kita dapat menghitung frekuensi natural (ωn) dan faktor redaman (ζ) sistem. Frekuensi natural (ωn) diperoleh dengan rumus;

$$\boldsymbol{\omega n} = \sqrt{\frac{\mathbf{a0}}{\mathbf{a2}}} = \sqrt{\frac{\mathbf{0.02206}}{\mathbf{1.5 \times 10^{-5}}}} = \mathbf{46.98}\, \boldsymbol{rad/s}$$

$$\boldsymbol{\zeta} = \frac{\boldsymbol{\alpha 1}}{\mathbf{2}\sqrt{\boldsymbol{\alpha 2 \alpha 0}}} = \frac{\mathbf{0.003066}}{\mathbf{2}\sqrt{\mathbf{1.5.10^{-5}} \times \mathbf{0.02206}}} = \mathbf{0.567}$$

This equation describes the dynamic response of the system to an input in the Laplace domain. Thus, this transfer function provides an overview of how the system will respond to a specific input signal, determined by the previously calculated natural frequency and damping factor.

Modeling a motor in close loop circuit

In this closed-loop circuit, what differentiates it from the open-loop circuit is the presence of a feedback plan. Mathematically, it can be said that the output of the transfer function is a real number, whereas the feedback is an imaginary number. Values need to be added, from which the rise time value will emerge. In the case of a rotary motor, the feedback value can be obtained by multiplying the transfer function by ϴ(s). In the transfer function that I have obtained with units in rad/s, this transfer can be multiplied by 1/s to convert it to radians. The equation is as follows:

$$Gs = \frac{0.09}{1.5 \times 10^{-5}\, S^2 + 0.003066\, S + 0.02206} \times \frac{1}{S}$$

$$Gs = \frac{0.09}{1.5 \times 10^{-5}\, S^3 + 0.003066\, S^2 + 0.02206\, S + 0}$$

Before that, a rise time value is needed to be used as a reference point/reference value.

Once this value is obtained, we can proceed to the above step. After obtaining the value with the modified transfer function, that will become the feedback value. In this closed-loop circuit, what differentiates it from the open-loop circuit is the presence of a feedback plan. Mathematically, it can be said that the output of the transfer function is a real number, whereas the feedback is an imaginary number.

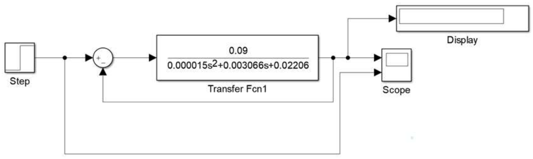

Fig. 6 *Close loop Control System*

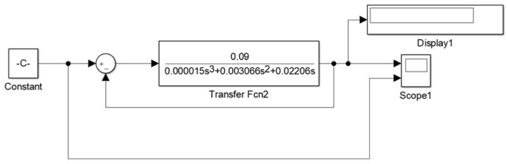

Fig. 7 *Close loop Control System with the modified transfer function*

2.Discussion

Based on the findings:

1. The proportional (P) component

Produces an output that is proportional to the current error, which is the difference between the setpoint (desired value) and the process value (measured value). The purpose of the proportional function is to directly reduce the error, with larger errors resulting in larger corrections. However, this component alone cannot eliminate steady-state error or constant error.

2. The integral (I) component

Calculates the accumulated error over time and adjusts the output to eliminate residual errors that may not be corrected by the proportional component alone. By summing past errors, the integral function can eliminate steady-state errors, although this may slow down system response and increase the risk of overshoot.

3. The derivative (D)

Component responds to the rate of change of error by predicting error trends and taking corrective action to reduce overshoot and improve system stability. The derivative function dampens rapid changes in error and speeds up system response, although it is sensitive to noise in the error signal which can cause undesirable output fluctuations.

4. Effectiveness of PID in Open Loop:

The research indicates that the PID method is optimal for open-loop control systems. PID controllers are versatile in adjusting control inputs based on proportional, integral, and derivative terms. This capability allows them to effectively regulate systems without feedback, ensuring accurate and stable performance across varying conditions and disturbances.

5. Advantages of P Controller in Closed Loop:

In closed-loop systems, the P controller has been identified as favorable due to its ability to achieve a fast rise time and minimal overshoot. The parameters $\mathrm{Kp} = 4.5$ and critical period $\mathrm{Pcr} = 38.3458$, determined using the Routh-Hurwitz method, highlight the controller's capacity to quickly approach and stabilize around the desired setpoint. This makes the P controller particularly suitable for applications where precise control with minimal oscillations is crucial, such as in robotic systems or industrial processes requiring rapid response to input changes.

6. Comparison and Practical Applications:

Comparing the two methods reveals that PID is beneficial in scenarios where precise open-loop control is essential, offering robustness and adaptability. On the other hand, the P controller excels in closed-loop systems by providing a faster response time and tighter control over the system's behavior, minimizing deviations from the desired output without introducing excessive oscillations.

In summary, while PID controllers are well-suited for maintaining stability and accuracy in open-loop environments, the P controller, with its optimized parameters derived from rigorous analysis methods like Routh-Hurwitz, proves effective in achieving responsive and stable closed-loop control. These insights guide the selection of appropriate control strategies tailored to specific operational requirements and performance criteria in engineering applications.

IV.CONCLUSION

Based on the research findings, the PID method has proven effective for implementing control in open-loop systems. Meanwhile, for closed-loop systems, the approach using a P controller shows fast rise time and minimal overshoot. Using the Routh-Hurwitz method to determine the parameters of the P controller yielded $\mathrm{Kp} = 4.5$ and a critical period value $\mathrm{Pcr} = 38.3458$.

These conclusions indicate that the PID approach is suitable for open-loop applications, while the P controller approach with parameters derived from Routh-Hurwitz analysis can provide quick and stable responses in closed-loop systems, optimizing rise time and reducing overshoot.

V.REFERENCE

1.Acknowledgement

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