37GB500-72-2463 DC Motor Analysis with Linear Quadratic Regulator Approach and Linear Quadratic Tracking

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ABSTRACT

In DC motors, it is an electrical device that consumes DC electrical power to produce mechanical torque. Because they are capable of producing relatively high torques to drive loads of the same size, DC motors are often used in a variety of applications. Permanent magnet motors, on the other hand, are linear, while DC motors are non-linear. Applications that require automatic speed control tend to have difficulty overcoming the non-linear nature of DC motors. The non-linear dynamic model of DC motors has constraints in the design of closed feedback loop control. Factors such as saturation and friction can affect the performance of conventional controls. DC motors are widely used in a variety of industries because they are able to handle applications with a wide range of power requirements, both in fixed-speed and variable-speed drives. In this paper, the analysis of the temporary speed parameters of DC motors was carried out separately using the Linear Quadratic Regulator (LQR) method. The DC motor speed of the three different motors with diverse specifications was analyzed by the LQR method. The simulation results using MATLAB software show that motors with low power applications show better and significant response with the application of the LQR method. This includes the ability to minimize deviations in speed, simple design, and reduced costs, which makes it efficient for use in controlled and controlled systems. These properties allow the speed control of DC motors to achieve the desired performance in systems with various power requirements. In addition to LQR, Linear Quadratic Tracking (LQT) is also used to control translational movements. LQT parameters are obtained through Genetic Algorithm (GA).

Keywords: *DC motor, system optimization, control system, Linear Quadratic Regulator, Linear Quadratic Tracking, Matlab and Mathematical Model.*

I. INTRODUCTION

In today's era, control optimization has become a very interesting topic due to the increasing need for high-performance systems. In the concept of control system optimization, there is a selection of performance and engineering indices that will lead to the development of an optimal control system within the limits of physical constraints. In order to achieve an optimal control system, the main goal is to find decision-making rules that will result in as much reduction as possible from performance deviations to the ideal value[1][2].

The author in this report discusses the Linear Quadratic Regulator (LQR) method which is part of the PPNS Automation Engineering course, specifically "System Optimization." LQR is one of the most effective control methods for state-based systems. The LQR controller has two parameters, a weight matrix Q and R[3][4]. These parameters must be specified to produce the expected control action. Examples of use of the LQR method include induction motor speed regulation, frequency control on electric generators, and drone quadcopters[5][7]. The LQR method has become very important to be combined with the discipline of system optimization to find the optimal point and reduce errors in a tool so that we can set the work of a tool to our own liking[8].

The need for electric motors is a basic need in driving industrial progress[9][10]. The projected demand for electric motors worldwide is expected to grow by 6.5%

annually. The Asia/Pacific region has the largest share of sales in this regard. This data shows that electric motors, including DC motors, play a key role in improving production speed and quality in the industry.

However, often the use of DC motors in industrial environments poses challenges related to torque regulation[11]. The researchers also suggested that measuring torque variables in DC motors is often difficult, so they are forced to estimate these torque variables.

DC motors operate non-linearly, especially when subjected to load variations[12]. Conventional controllers, such as PID controllers, are generally used because they have good performance in controlling linear systems[13]. In the context of induction motor control, a PID controller with a field-oriented control (FOC) method is used to improve the torque current control signal in the PID controller[14].

In addition, some researchers have applied optimal control techniques, such as using Linear Quadratic Regulators (LQR)[15]. In this paper, a DC motor with a permanent magnet is used as a plant that will be identified and controlled by the optimal control method of LQR[16]. This research also involves the development of a DC Motor identification module using Simulink-Matlab. The value of the feedback gain and the performance index of LQT depend on the combination of the Q and R matrices.

To get the optimal combination of the Q and R matrices, manual tuning methods are often used (trial and error). However, often the results obtained are not optimal. Therefore, in this study, the Genetic Algorithm (GA) method is used as an alternative to replace manual tuning.

II. METHODOLOGY

2.1 Switch Function

Function transfer is a comparison between the output Laplace function and the input Laplace function, assuming all initial conditions are considered zero. Function switching is used to make it easier to understand the characteristics of a system. In general, the form of the 1st order system can be explained as follows

$$G(s)=\frac{K}{\tau s+1} \quad (1)$$

Meanwhile, the standard form of the 2nd order system can be explained as follows

$$G(s)=\frac{\omega\pi^2}{s^2+2\varsigma\omega ns+\omega\pi^2} \quad (2)$$

2.2 Linear Quadratic Regulator

In contemporary control theory, the linear quadratic regulator (LQR) approach is used. The space-state method is used to conduct this type of system analysis[17]. This method is used by multi-input multi-output systems because of the simplicity of the country space method.

A common use of linear quadratic controller design techniques is in applications involving DC motor control systems with LQR controllers. In this system, the speed of the motor can change due to variations in load and other external disturbances. The main purpose of using LQR is to minimize deviations in motor speed. This system compares the speed of the motor, which is the output of the system, with the input provided to the system, i.e. the voltage of the motor, Reduction of deviations in motor speed is done with the help of LQR

$$x=Ax+Bu$$

$$y=Cx \quad (3)$$

The performance index of the minimum energy (costfunction/quadratic function) is shown by Equation (2)

$$J=\frac{1}{2}\int_{0}^{\infty}(x^{T}\text{¿}Q+u^{T}Ru)dt\,\text{¿} \qquad (4)$$

The regulator equation can be solved by solving the Riccati algebraic equation according to Equations (5)-(7).

$$A^{T}P+PA-PBR^{-1}B^{T}P+Q=0 \qquad (5)$$

$$-K=R^{-1}B^{T}P \qquad (6)$$

$$u=-Kx \qquad (7)$$

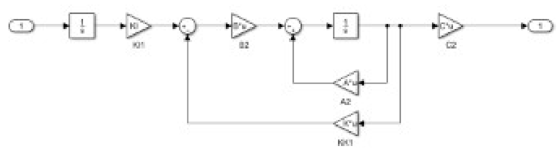

Figure 2.1 Linear Modeling of Quadratic Regulators

2.3 Linear Quadratic Tracking

The Quadratic Tracking linear management system has a system output that follows the desired reference (trajectory)[18]. The equation state (3) and the error vector, as shown in Equation (4), exist in a system. Equation (5) also describes the performance index of the system.

$$x=Ax+Bu$$

$$y=Cx \qquad (8)$$

$$e=z-y \qquad (9)$$

$$J=\frac{1}{2}e(t_{F})F(t_{f})e(t_{f})+\frac{1}{2}\int_{t_{0}}^{t_{f}}[eQe+URu]dt \qquad (10)$$

After obtaining the mathematical model of the system in the form of state-space, Equation (11) can be used to obtain the solution matrix of Riccati's differential equations for the infinite time case [20].

$$0=-PA-AP+PBR^{-1}BP+CQC \qquad (11)$$

The desired performance of the system can determine the Q and R matrices. Equation (12) can be used to find a nonhomogeneous vector differential equation after obtaining the Riccati equation.

$$g_{(12)}=-\left[A-BR^{-1}BP\right]g-CQz \qquad (12)$$

After obtaining the matrix P, which is a positive definitive symmetrical matrix, we can find the value of the feedback gain K using Equation (13)

$$K=R^{-1}BP \qquad (13)$$

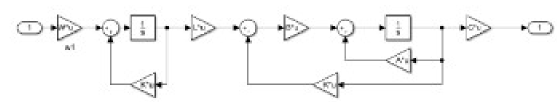

Figure 2.2 Linear Modeling of Quadratic Regulators

2.4 Noise

In a communication system, noise is interference in the form of unwanted signals that are always present in the transmission system[19]. The presence of this noise can disrupt the quality of the desired signal received, and ultimately, can interfere with the process of transmitting and receiving data.

2.5 DC Motor Modeling

2.5.1 LQR Order 1 DC Motor

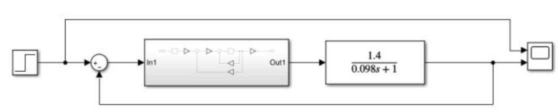

Figure 2.5.1 LQR Modeling Order 1 DC Motor

2.5.2 LQT Order 2 DC Motors

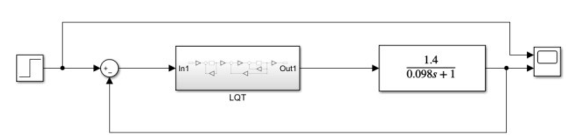

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Figure 2.5.2 LQR Modeling Order 2 DC Motor

2.5.3 LQR Order 1 DC Motor with Noise

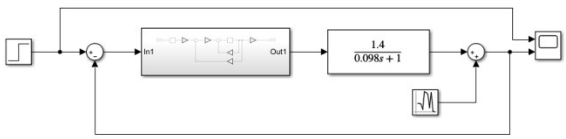


Figure 2.5.3 LQR Modeling Order 2 DC Motor with Noise

2.5.4 LQT Order 1 DC Motor with Noise =

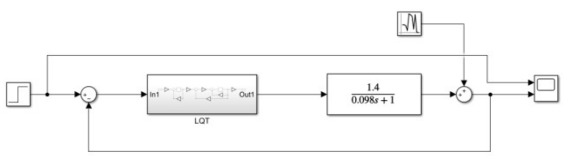


Figure 2.6 LQR Modeling Order 2 DC Motor with Noise

2.6 DC Motor Modeling

Customizable Parameters :

1. Length A,B and L

2. Electric Performance

3. Direction of rotation

4. Reduction Ratio

L(mm) Gearbox Length "L"	Stage Number	Reductin Ratio							
18.5	2	10.3/1							
21.5	3	19.5/1	31/1						
24	4	40/1	51.5/1	52/1	72/1	93/1			
26.5	5	155/1	168/1	196/1	217/1	278/1			
29	6	360/1	371/1	464/1	557/1	650/1	835/1		

型号 MODEL	电压 VOLTAGE		无负荷 NO LOAD		最大效率点 AT MAXIMUM EFFICIENCY					堵转 STALL		
	使用范围 OPERATING RANGE	额定值 NOMINAL	转速 SPEED	电流 CURRENT	转速 SPEED	电流 CURRENT	转矩 TORQUE		功率 OUPUT	转矩 TORQUE		电流 CURRENT
		V	rpm	A	rpm	A	kg.cm	mN.m	W	kg.cm	mN.m	A
37GB500-72-2463	8.0~24.0	24.0	63	0.019	49.1	0.07	1.00	98.14	0.50	4.56	447.50	0.16
37GB500-93-1256	5.0~14.0	12.0	59	0.042	47	0.15	1.60	157.02	0.77	7.60	745.83	0.57
37GB500-835-0603.2	3.0~8.0	6.0	3.2	0.045	2.5	0.075	6.00	588.81	0.15	24.60	2414.13	0.25
37GB365A-155-1256	5.0~14.0	12.0	56	0.15	44.8	0.62	5.00	490.68	2.30	25.50	2502.45	2.4
37GB360A-371-2418	8.0~26.0	24.0	18	0.076	14	0.23	10.00	981.35	1.44	13.00	1275.76	0.74
37GB365H-62-12125	5.0~14.0	12.0	125	0.13	100	0.42	2.00	196.27	2.05	10.15	996.07	1.62

Figure 2.6 Specification of DC Motor 37GB500-72-2463

- Motor Name = 37GB DC Motor500-72-2463
- T = 98.14 mNm = 0.098 N/m
- Current = 70 mA = 0.07 A
- Voltage = 24V
- Speed = 270.91 m/s
- ς = 10.1
- Resistance = 342.85 ohm
- Inductance = 0.00001805 H
- Moment of Inertia = $\frac{1}{2} \times m\left(R1^2+R2^2\right)$

$$= \frac{1}{2} \times 0{,}032\left(11^2+11^2\right)$$

$$= 3{,}87\text{kg} \; \backslash m^2$$

2.7 Order 1 and Order 2 Mathematical Models

2.7.1 Order 1

General forms of order 1 transfer functions

$$G(s)=\frac{K}{\tau s+1} \tag{14}$$

Order 1 DC Motor Based on the DC motor datasheet, the order 1 equation is obtained Where

$$\tau = K.i$$

So that

$$\tau = 0{,}098\,Nmi = 0{,}07\,A$$

General equations of the 1st order:

$$\tau = K.i$$

$$K=\frac{\tau}{i}$$

$$K=\frac{0{,}098}{0{,}07}=1{,}4$$

Information:
K = Coefficient of DC motor
 = DC motor torque
I = DC motor current
So that the order 1 equation of DC motors is obtained:

$$G(s)=\frac{K}{\tau s+1}$$

$$G(s)=\frac{1,4}{0,098\,s+1}$$

2.7.2 Order 2

The general form of the 2nd order transfer function is as follows:

$$G(s)=\frac{\omega n^2}{s^2+2\varsigma\omega ns+\omega n^2}$$

Based on the DC motor datasheet, the 2nd order equation on the dc motor is obtained as follows:
Known:

$$rasio\, redaman=10{,}3:1\;\omega=2f=2\times 3{,}14\times 50\, Hz$$

Information:

$$\omega n^2=frekuensi\, natural$$
$$\varsigma=Rasio\; Redaman$$

(17)

2.8 Script Matlab Linear Quadratic Regulator

```
% OPTIMIZATION OF LQR SYSTEM ON DC MOTORS
Clear;
CLC;
% DC Motor Models
J = 3.872 ; b= 10.1 ; K= 1.4 ; R= 342.85 ; L = 0.00015 ;
% J = Momeninesia , b = Ratiodam, K = constant, R = resistance, L=
% Inductance
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0]

AA = [ A zeros(2,1); -C 0];
BB = [B; 0];

% Pole Placement
J = [-3 -4 -5];
K = acker(AA,BB,J);
KI = -K(3);
KK = [K(1) K(2)];

% Matrix LQR
Q = [1 0 0;
   0 1 0;
   0 0 1000];
R = [1];

K_lqr = lqr(AA,BB,Q,R)
KI2 = -K_lqr(3);
KK2 = [K_lqr(1) K_lqr(2)];
```

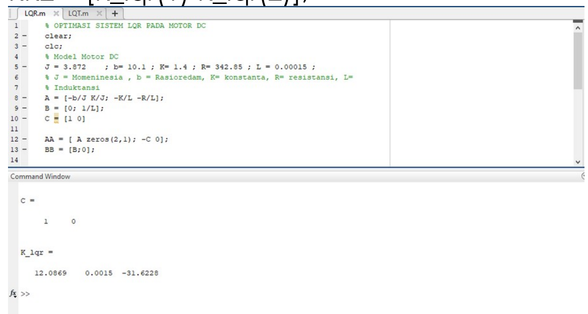


Figure 2.8 Running Results of LQR Matlab Script

2.9 Matlab Linear Quadratic Tracking Script

```
Clear;
CLC;
% DC Motor Models
J = 3.872 ; b= 10.1 ; K= 1.4 ; R= 342.85 ; L = 0.00015 ;
% J = Moment, b = Ratio, K= constant, R= resistance, L=Inductance
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0]
Q=10; R=0.0000000001; %0.00000000000001
W=C'*Q; %
[S,o,m,n]=care(A,B,C'*Q*C,R) %m=v(t) %S=P
K=inv(R)*B'*S %feedback Gain
ACL=(A-B*K)'
L=inv(R)*B' %model following gain
```

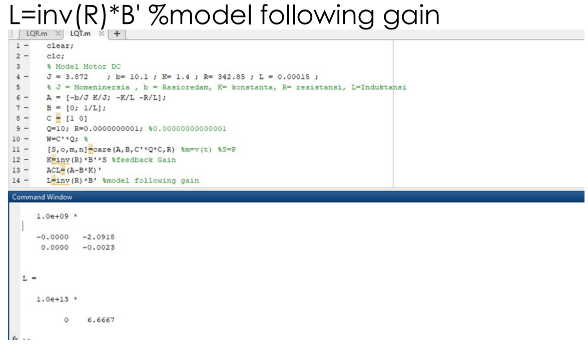


Figure 2.9 Runnning Results from LQT Matlab Script

III. RESULTS & DISCUSSION

In the simulation results, the input and output values of the two feedback methods, namely Linear Quadratic Regulator and Linear Quadratic Tracking, can be observed. Each experiment produces mixed results because each experiment uses a different Simulink configuration within MATLAB. Some results in MATLAB even exceed the calculation limits, making it difficult to determine the exact results of the simulation. In addition, in some simulations that use noise, the graph is difficult to interpret because the variation of input in the simulation is very diverse.

Information:

A = TUK SISO

B = TUK SISO with NOISE

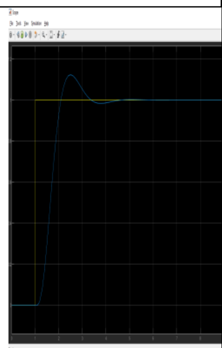
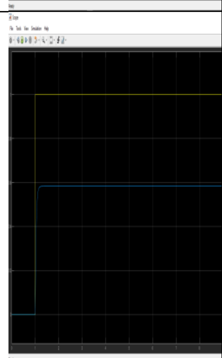
Method	Scape	No noise	
		Set point	Undershoot
LQR		1	-0.505%
LQT		1	-

Table 3.1 results of LQR and LQT simulations

From the results seen on the scope display, it can be concluded that the output of the 1st order DC motor system using the Linear Quadratic Regulator (LQR) method has succeeded in reaching the desired set point, even though it initially passes or falls below the set point of about -0.505%. On the other hand, the 1st order DC motor system that uses the Linear Quadratic Tracking (LQT) method does not reach the desired set point. The green and yellow graph lines depict the input values from the step input, while the blue graph lines depict the output of the 1st order DC motor system using the LQR and LQT methods.

3.2 DC Motor Simulation Results with Noise

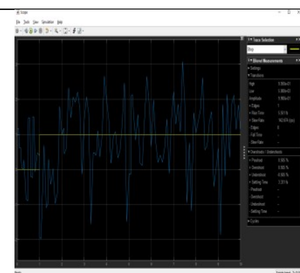
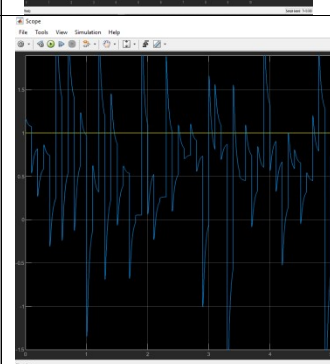
Method	Scape	No noise	
		Set point	Undershoot
LQR		1	-0.505 %
LQT		1	-0.505 %

Table 3. 2 SIMO and SIMO Noise Simulation Results

The results seen on the scope display show that the output of the DC motor system order 1 with the use of the Linear Quadratic

Regulator (LQR) and Linear Quadratic Tracking (LQT) methods has succeeded in reaching the desired set point, but is accompanied by additional input noise interference that causes the graph to become irregular. The yellow graph depicts the input value of the step input, while the blue graph depicts the output of the 1st order DC motor system using the LQR and LQT methods with input noise interference.

IV. CONCLUSION

The conclusion that can be drawn from the simulation using MATLAB Simulink software is that a transfer function derived from 1st order DC motor modeling is needed to analyze the step response of the LQR & LQT system of DC motors. To obtain a mathematical model of a DC motor of the 1st order, the data contained in the 37GB500-72-2463 DC Motor datasheet is very necessary. In this experiment, data from the 37GB500-72-2463 DC Motor datasheet was used. The simulation results of the LQR & LQT system, which utilizes the transfer function obtained from the mathematical modeling of 1st order DC motors using the 37GB500-72-2463 DC Motor datasheet, show that the system is able to achieve the output according to the desired set point.

V. CLOSING

1. Awards

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