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DC Motor VD-49.15-K1-B00 Using Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT)

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ABSTRACT

The results of this research show that the problem of optimal control is currently attracting increasing attention, especially due to the increasing demand for high-performance systems. The concept of control system optimization influences the selection of performance indicators and techniques used to create an optimal control system. To achieve an ideal control system, rules are needed that allow decisions to be made regarding control with the aim of minimizing deviations from ideal behavior. The Linear Quadratic Regulation (LQR) method is one of the optimal control approaches for state space-based systems. The LQR controller has two parameters, namely the weight matrices Q and R, which must be determined for optimal control according to expectations, as in the case of a quadcopter or four-wing drone.

Linear Quadratic Tracking (LQT) is a control system in which the output is set to follow a predetermined path through the input. In this example, we carry out a simulation using Simulink MATLAB with an LQT circuit that can be implemented in the MATLAB application. During the course "System Optimization," the author discusses the application of methods for manufacturing DC motors with LQR and LQT settings using data sheets. The data sheet was imported into a MATLAB script and then simulated using MATLAB Simulink software to see the results step by step. The DC motor used in this research is type VD-49.15-K1-B00 which is equipped with values for moment of inertia, motor constant, damping coefficient, resistance and inductance.

Key word : DC Motor VD-49.15-K1-B00, control, LQR, LQT,

I. INTRODUCTION

Today, optimal control has become a significant focus of attention, especially as the need for high-performance systems increases. The concept of optimization in control systems involves selecting performance indices and engineering techniques that will produce a control system that meets expectations within existing physical constraints [1]. In an effort to achieve an ideal control system, the main goal is to find decision-making rules in the control system that will minimize deviations from the desired behavior [2].

In this report, we will discuss the Linear Quadratic Regulator (LQR) method which is included in the "System Optimization" course material at PPNS. LQR is an optimal control method based on state space formulation. The LQR controller has two main parameters, namely the weight matrices Q and R, which must be chosen appropriately to achieve optimal control action according to expectations[3][4]. This LQR method has a variety of applications, from controlling the speed of induction motors, controlling the frequency in generator power plants, to controlling quadcopter drones or four-wing drones [5]. The integration of the LQR method is very important in the field of system optimization because it can help reach the optimum point and reduce errors in various equipment, so that the operation of the equipment can be adjusted to our needs [6][7].

Apart from the LQR method, we will also discuss Linear Quadratic Tracking (LQT), which is a system setup method where the output is set to follow a predetermined path through the given input [8]. In this case, we will carry out a simulation using MATLAB Simulink software to see the step response of the DC motor system by entering data from the available datasheet. The DC motor used is type VD-49.15-K1-B00 which has information regarding moment of inertia, motor constant, damping ratio, resistance and inductance which will be entered into MATLAB software for simulation.

II.METHODOLOGY

1. Research Process

In the context of a control system, the system identification process usually follows the steps as seen in Figure 1. The system identification process consists of four main stages:

1. Availability of input-output data from the plant system to be identified.
2. Selection of the model structure to be used.
3. Estimate the appropriate model parameters.
4. Validate the model that has been identified, including the structure and values of the parameters used.

The system identification process can be depicted in a block diagram as shown in Figure 1.

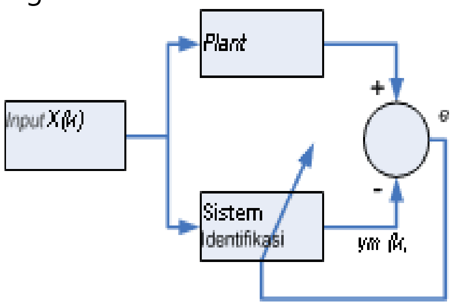


Figure 1. Block diagram of the identification process of a system.

In this research, DC motor system identification was carried out in open loop mode using an Arduino device and a personal computer (PC) connected to the Simulink-Matlab device [9][10]. The identification method applied is a static identification method.

The commonly used static identification method is to use the open loop method and provide step input, as shown in Figure 2.

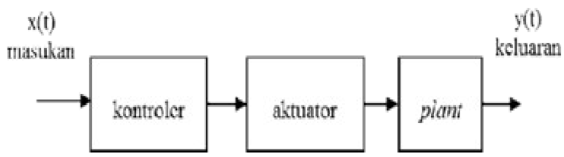


Figure 2. Block diagram of plant identification using the open loop method

Next, this research focuses on system characteristics. System characteristics reflect the dynamic behavior of the system, and are often referred to as system performance specifications[11]. The system output response appears after an input signal or test signal is given. System response characteristics can be grouped into time response characteristics and frequency response characteristics. In this study, we focus attention on the time response characteristics of DC motors[12].

The purpose of observing time response characteristics is to understand how the system output response changes over time. In general, time response performance specifications can be divided into two observation stages, namely transient response specifications and steady state response. Furthermore, observations of system characteristics can be approached with first-order and second-order system models.

2. Linier Quadratic Regulator

The best system is a system that can achieve optimal performance according to the references provided[13]. To achieve optimal system control, optimization criteria are needed that are able to minimize the difference between the system's actual behavior and ideal behavior when the system experiences deviation[14].

This difference is measured through the performance index, which is an evaluation parameter that shows the extent to which the system performance corresponds to the desired one[15]. The performance index serves as a marker for optimal control, with lower performance index values indicating a more optimal system[16][17]. In some cases, disturbances can cause changes in the controlled variables, and the controller must be able to compensate for these disturbances.

Linear Quadratic Regulator (LQR) is a technique used to design optimal control systems. One of the advantages of the second-order optimal control method is its ability to systematically produce a state feedback gain matrix (K) for a system with m inputs (u)[18]. The control signal format can be described as follows:

$$(t) = -K\,x(t) \tag{1}$$

Index work:

$$J = \int \infty [x^T Q x + u^T R u]\, dt \tag{2}$$

Parameter:

Q = matriks simetris, semi definit positif, real (Q≥0).

R = matriks simetris, definit positif, real (R>0).

The Q and R matrices have an important role in determining the error rate and energy consumption in this context. In this case, we assume that the control vector u(t) is infinite. The linear control law contained in equation (1) is considered the optimal control law. The form of equation (1) is most appropriate when we have uncertainty in the elements of the matrix K and have determined the minimum number of preferences.

3. Linier Quadratic Tracking

The application of optimal control theory to linear systems has many applications in industry and education. This optimal control approach aims to reduce energy consumption by optimizing the function of the energy used, thereby producing optimal performance[19]. One application of this concept is in the vehicle suspension system, which aims to reduce the effects of shocks or disturbances when the vehicle crosses uneven roads.
One of the problems overcome is the steady state error by using an integral controller[20]. The integral controller works by calculating the integral output

error as an additional state variable in the system.
The aim of control in this research is to control the position of the vehicle through the suspension system in such a way that the vehicle remains in the desired position with a minimum level of steady state error when shocks or disturbances occur. The controller used in this research is the Linear Quadratic Integral Tracking (LQIT) method, which is a modification of the Linear Quadratic Tracking (LQT) controller with the addition of an integral controller.
Linear Quadratic Regulator (LQR) is an optimal control method for linear systems with quadratic criteria used to solve regulator problems. Meanwhile Linear Quadratic Tracking (LQT) is an optimal control method for linear systems with quadratic criteria used to solve tracking problems. The equation of state of a general system is represented in a form that describes the relationship between various variables in the system.

$$x = Ax + Bu \quad (3)$$
$$y = Cx \quad (4)$$

Parameter,
$x_{n*1} = (state)\, system$
$U_{m*n} = State\ input$
$y_{1*1} = State\ output$
A = Matriks sistem
B = Matriks sistem
C = Matriks output
By minimizing energy (cost function/quadratic function) through performance indices in intervals [t_0,∞] :

$$J = \frac{1}{2} \int_{t_0}^{\infty} \left(e^T Q e + u^T R u \right) dt \quad (5)$$

Dengan
t_0 = waktu awal
∞ = waktu akhir
Q = matriks semidefinite positif
R = matriks definit positif

Persoalan regulator dan tracking dapat diatasi atau diselesaikan dengan Persamaan Riccati sebagai berikut:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \quad (6)$$

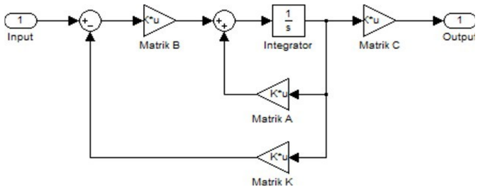


Figure 3. LQR circuit

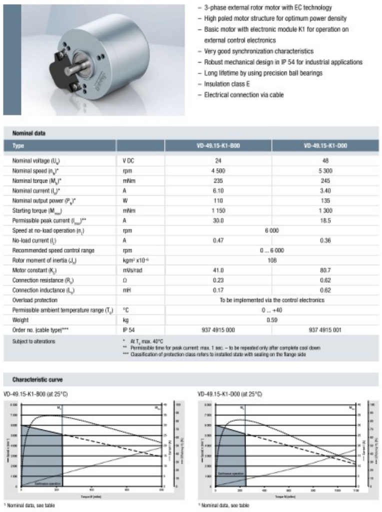
In designing LQR and LQT controllers, the selection of the weight matrix is of key importance. The greater the Q value, the closer the system is to its minimum point, which results in minimal energy use. The optimal control block diagram can be seen in Figure 1. The main goal is to ensure that the value of the solution to the Riccati Equation becomes a matrix that has small values, such as:

$$K = R^{-1} B^T P \quad (7)$$
$$u = -Kx \quad (8)$$

4.Noise

Noise or disturbance is a signal that has the potential to influence the output value of a system. Disturbances originating from within the system are called internal disturbances, while disturbances originating from outside the system are called external disturbances. The presence of this noise can cause the system output value to become unstable or not as desired.

5. DC Motor Specifications



- 3-phase external rotor motor with EC technology
- High poled motor structure for optimum power density
- Basic motor with electronic module K1 for operation on external control electronics
- Very good synchronization characteristics
- Robust mechanical design in IP 54 for industrial applications
- Long lifetime by using precision ball bearings
- Insulation class E
- Electrical connection via cable

Nominal data

Type		VD-49.15-K1-B00	VD-49.15-K1-D00
Nominal voltage (U_N)	V DC	24	48
Nominal speed (n_N)*	rpm	4 500	5 300
Nominal torque (M_N)*	mNm	235	245
Nominal current (I_N)*	A	6.10	3.40
Nominal output power (P_N)*	W	110	135
Starting torque (M_{max})	mNm	1 150	1 300
Permissible peak current (I_{max})**	A	30.0	18.5
Speed at no-load operation (n_L)	rpm	6 000	
No-load current (I_L)	A	0.47	0.36
Recommended speed control range	rpm	0 ... 6 000	
Rotor moment of inertia (J_R)	kgm² x10⁻⁶	108	
Motor constant (K_E)	mVs/rad	41.0	80.7
Connection resistance (R_V)	Ω	0.23	0.62
Connection inductance (L_V)	mH	0.17	0.62
Overload protection		To be implemented via the control electronics	
Permissible ambient temperature range (T_U)	°C	0 ... +40	
Weight	kg	0.59	
Order no. (cable type)***	IP 54	937 4915 000	937 4915 001

Subject to alterations
* At T_U max. 40°C
** Permissible time for peak current: max. 1 sec. – to be repeated only after complete cool down
*** Classification of protection class refers to installed state with sealing on the flange side

Characteristic curve

VD-49.15-K1-B00 (at 25°C) VD-49.15-K1-D00 (at 25°C)

Figure 4. Datasheet for DC motor type VD-49.15-K1-B00

Motor Specifications

τ	= 235 mNm/A
	= 0,235 N/m
No load current	= 0.47 A
Rated Current	= 6.10 A // arus
Voltage	= 24 V
Speed	= 6000 rpm atau 628m/s
Diameter	= 48 mm
Radius Motor	= 24 mm = 0,024 m
Damper ratio	= 108 kgcm^2 x 10^{-6}

General form of 1st order transfer function:

$$G(s)=\frac{K}{\tau s+1} \quad (9)$$

1st Order DC Motor
Based on the DC Motor datasheet, the 1st order equation is obtained:

$$\tau = K.i$$

$$K=\frac{\tau}{i}=\frac{0{,}235}{6{,}10}=0{,}0385 \quad (10)$$

1st Order DC Motor:

$$G(s)=\frac{0{,}0385}{0{,}235\,s+1} \quad (11)$$

6.Matlab

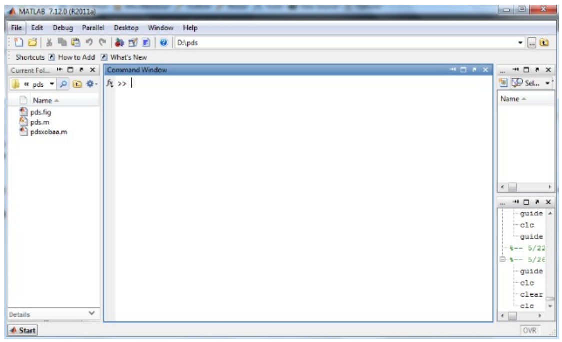


Figure 5. MATLAB display

MATLAB is a programming platform based on a matrix-oriented language, and is often used to analyze data, develop algorithms, and create modeling and applications. The MATLAB software interface looks as shown in Figure 5.
In order to carry out a simulation to observe the response of a DC motor, the author uses the Simulink feature available in the MATLAB tool. Simulink is one of the main components in MATLAB which is specifically designed for graphical programming.
Simulink has a major role in creating dynamic system simulations. This simulation process is carried out using a functional diagram consisting of various blocks, each with relevant functions and connected in an equivalent way. Simulink has the main function as a modeling, simulation and dynamic system analysis tool, all of which can be accessed via a graphical user interface. Simulink also includes a diverse collection of tools that can be used to analyze both linear and non-linear systems.

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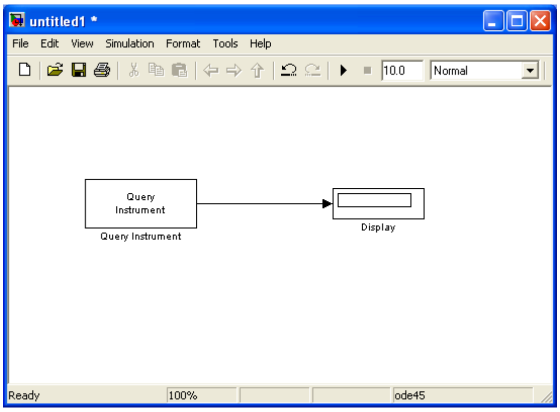
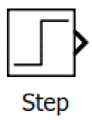
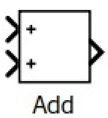
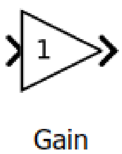
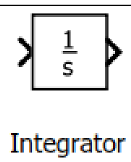


Figure 6. MATLAB Simulink display

7.Tool Simulink

Tabel 2. Component Simulink

Component	Name
	Step
	Random Number
	Add
	Gain
	Integrator
	Scope

8.Script

1.LQR

```
% OPTIMASI SISTEM LQR PADA MOTOR DC
clear;
clc;
% Model Motor DC
J = 360    ; b= 0.01 ; K= 0.056 ; R= 0.70 ; L = 0.00049 ;
% J = Momeninersia , b = Rasioredam, K= konstanta, R= resistansi, L=Induktansi
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0]

AA = [ A zeros(2,1); -C 0];
BB = [B;0];

% Pole Placement
J = [-3 -4 -5];
K = acker(AA,BB,J);
KI = -K(3);
KK = [K(1) K(2)];
% Matrix LQR
Q = [1 0 0;
   0 1 0;
   0 0 1000];
R = [1] ;

K_lqr = lqr(AA,BB,Q,R)
KI2 = -K_lqr(3);
KK2 = [K_lqr(1) K_lqr(2)];
```

2.LQT

```
% OPTIMASI SISTEM LQT PADA MOTOR DC
clear;
```

```
clc;
% Model Motor DC
J = 360   ; b= 0.01 ; K= 0.056 ; R= 0.70 ; L
        = 0.00049 ;
% J = Momeninersia , b = Rasioredam, K=
        konstanta,      R=      resistansi,
        L=Induktansi
A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0]
Q=10;                   R=0.0000000001;
        %0.00000000000001
W=C'*Q; %
[S,o,m,n]=care(A,B,C'*Q*C,R)        %m=v(t)
        %S=P
K=inv(R)*B'*S %feedback Gain
ACL=(A-B*K)'
L=inv(R)*B' %model following gain
```

III. RESULT & DISCUSION

1. Simulink in LQR

1. LQR Circuit

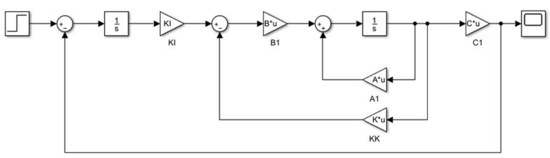


Figure 7. DC motor LQR simulink circuit

2. Simulink LQR DC Motor VD-49.15-K1-B00 circuit without noise

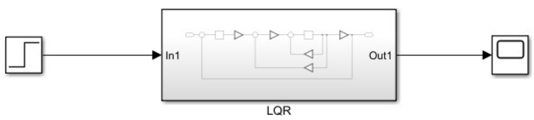


Figure 8. DC motor LQR simulink circuit without noise

3. Simulink LQR DC Motor VD-49.15-K1-B00 Circuit with Noise

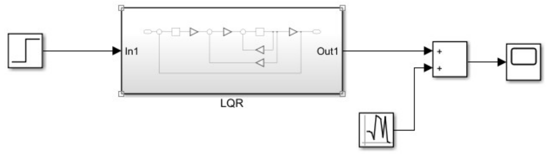


Figure 9. Simulink LQR DC motor circuit with noise

2. Simulink in LQT

1. LQT Circuit

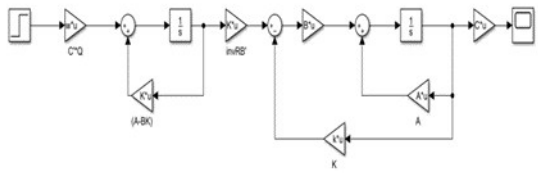


Figure 10. DC motor LQT simulink circuit

2. Simulink LQT DC Motor VD-49.15-K1-B00 series without noise

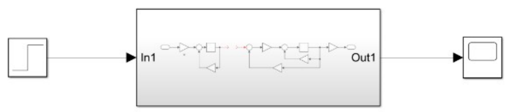


Figure 11. Simulink LQT DC motor circuit without noise

3. Simulink LQT DC Motor VD-49.15-K1-B00 circuit with noise

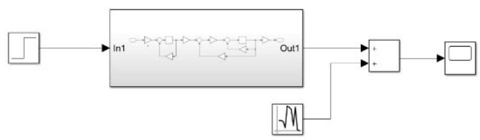


Figure 12. Simulink LQT DC motor circuit with noise

3. LQR Circuit Output Graph Without Noise

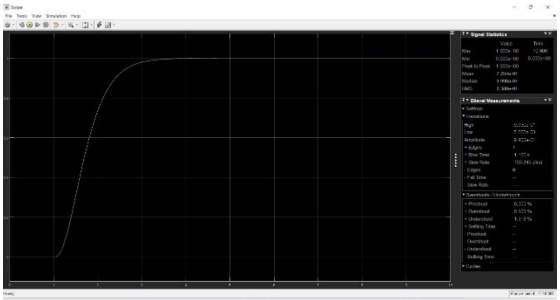


Figure 13. LQR Circuit Output Graph Without Noise

4. LQR Circuit Output Graph With Noise

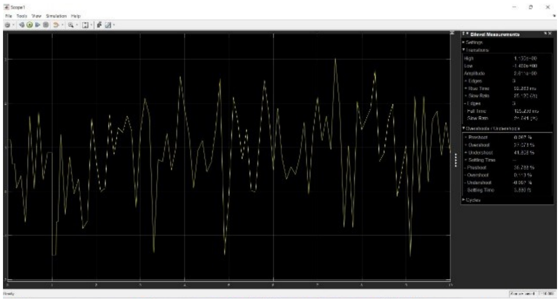


Figure 14. LQR Circuit Output Graph with Noise

5. LQT Circuit Output Graph Without Noise

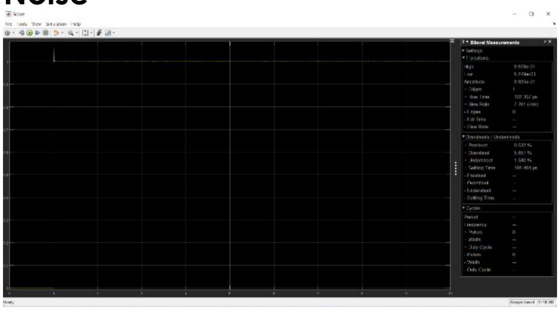


Figure 15. LQT Circuit Output Graph Without Noise

6. LQT Circuit Output Graph with Noise

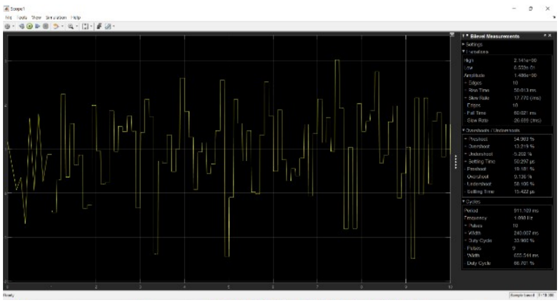


Figure 16. LQT Circuit Output Graph with Noise

7. Discussion

- LQR Without Noise In Figure 13 you can see the output response graph produced by the LQR circuit without any noise. Note that the response output of the VD-49.15-K1-B00 type DC motor without any disturbance reaches an amplitude almost reaching the setpoint value. In addition, the rise time for this response reaches a maximum value of around 1.2 seconds, with a low level of overshoot and undershoot, namely around 0.507%.

- LQR With Noise In Figure 14, you can see the output response in the LQR circuit which is influenced by noise in the VD-49.15-K1-B00 type DC motor. The graph shows fluctuations in the response output due to the influence of added noise (in the form of random numbers). The system in this condition has not yet reached the setpoint amplitude value, with a rise time of around 52.73 milliseconds, and an overshoot level of 102.93% and an undershoot of around -87.68%.

- LQT Without Noise Figure 15 shows the output response graph of the LQT circuit without noise on the DC motor plant VD-49.15-K1-B00. In this output graph, the amplitude reaches a value of around 9.924, and there is an overshoot level of around 5.59%. It can be seen that in the LQT circuit there is quite a large overshoot, in contrast to the LQR output graph which has a low level of overshoot, and is able to reach setpoint slowly and gradually.

- LQT with Noise In Figure 16 you can see the output response graph of the LQT circuit which is influenced by noise in the VD-49.15-K1-B00 DC motor plant. The LQT response graph with noise shows an undershoot of around -9.135% and an overshoot level of around 13.218%. This circuit also displays fluctuations in the output graph due to the influence of added noise, such as the use of random numbers. The response of this circuit does not reach setpoint, with a rise time of around 50.013 milliseconds.

IV.CONCLUSION

After carrying out a series of experiments using the LQR circuit with and without noise on a DC motor type VD-49.15-K1-B00, this LQR experiment involves a mathematical calculation process to find the 1st order parameters. This approach is based on the information contained in the datasheet BN28 brushless motor, which includes the values of moment of inertia, inductance, resistance, damping ratio and motor constant.

The results of the experiment show that at the system response output using the LQR circuit without any load, the VD-49.15-K1-B00 type DC motor reaches an amplitude that is close to the set point value, with a rise time of around 1.2 seconds. There are also very small changes in the form of overshoot and undershoot, namely around 0.508%.

Meanwhile, in the system response graph with the addition of noise in the LQR series, fluctuations can be seen caused by the presence of noise (random numbers). The system has not been able to reach the setpoint with an amplitude of around 0.68. The rise time is quite long, namely around 52.73 milliseconds, and there is an overshoot of 102.93% and an undershoot of around -87.68%.

Furthermore, in the system response graph using the LQT circuit without any noise, it can be seen that the amplitude reaches 9.924, but the system experiences an overshoot of 5.58%. This LQT response graph shows a greater overshoot than LQR, as well as a faster and more aggressive movement towards the setpoint.

However, in the system response graph using the LQT circuit in the presence of noise, there is an undershoot of around -9.135% and an overshoot of 13.218%. This circuit also shows fluctuations caused by the presence of noise, so it does not reach the set point, with a rise time of around 50.013 milliseconds.

V.REFERENCE

1. Acknowledgement

The researchers realized that it was difficult to create a magazine that could serve the community without the help and support of many parties.

Therefore, on this occasion the researcher would like to express his deepest gratitude to all parties who have participated and contributed to this project.

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