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Differential Optimization Control for MG-16B DC Motor with LQR and LQT Circuits

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ABSTRACT

DC motors are an electronic component that is very often found in various daily applications. Basically, DC motors have a tendency to experience changes in speed when exposed to load, so the speed will not tend to remain constant. So that the speed of a DC motor remains stable when the load changes, a special controller is needed. One of the control methods used to improve DC motor performance is through the use of LQR (Linear Quadratic Regulator) and LQT (Linear Quadratic Tracker). LQR aims to bring the motor response closer to the desired setpoint value, as well as reducing overshoot and undershoot symptoms that can occur in the system. On the other hand, LQT is a linear control method that allows the system output to follow the desired setpoint value. The LQR method ensures that the motor response reaches setpoint without any overshoot or undershoot. Without implementing the LQR method, the motor response will be far from the desired setpoint value and it will take longer to achieve the desired motor response. By using the LQT method, the motor response is faster, in only about 0.5 seconds, but there may be slight fluctuations (ripples) in the response. Therefore, in controlling the MG-16B type DC motor, the LQT method is preferred over the LQR method because it is able to provide a faster response to reach the setpoint value.

Key Word : DC Motor, Noise, optimization, LQR, LQT

I. INTRODUCTION

DC motors are electronic devices that are widely used in various applications in everyday life. An electric motor is an electromagnetic device that converts electrical energy into mechanical or motion energy [1]. The operating principle of a DC motor involves a rotor with a magnetic field that is always in the opposite direction to the stator magnetic field, which causes a repulsive interaction between the two. The coil resists the flow of voltage, which creates a field in a certain direction around the armature coil [2][3]. Usually a DC motor will experience a decrease in speed when it bears additional load, so it cannot maintain a constant condition. The speed of a DC motor can be controlled by changing the input voltage [4]. To overcome this, the solution is to increase the input voltage of the DC motor to return its speed to the desired value. Therefore, the use of a controller is necessary to ensure that the DC motor speed remains stable in the face of changing loads. The basic function of control is to compare the actual value of the plant output with a reference input (set point), determine the error and generate a control signal that reduces the error to almost zero. [5] [6].

One of the control methods used to improve DC motor performance is by applying the LQR (Linear Quadratic Regulator) and LQT (Linear Quadratic Tracker) methods [7][8]. The main function of LQR is to approach the motor response to the desired point and at the same time reduce the effects of overshoot and undershoot that sometimes occur in the system [9]. This control approach has characteristics such as stability, reliability, and the ability to produce static gain. Primarily, this linear-quadratic control method is used to design optimal controllers that can minimize certain costs or optimize system efficiency indices[10].

LQT is a linear regulation system where the system output follows the desired reference (trajectory) [11]. Linear Quadratic Tracker (LQT) is a model-based tracking control mechanism that uses affine state feedback to achieve optimal control. LQT consists of a regular linear dynamic system mode feedback switch and an additional input control term. Feedforward expression depends on a reference signal vector [12]. Typically, LQT is used to optimize systems related to tracking and pathfinding.

In the context of experiments carried out in the Automation Engineering System Optimization Course at the Surabaya State Shipping Polytechnic, the research aims to evaluate the effect of the LQR and LQT methods on the output response produced by a DC motor[13]. The experiment also included adding noise to the system to assess whether the use of the LQR and LQT methods had an impact on the motor's response to the given noise disturbance, as well as to identify performance differences between the two optimization methods on DC motors of the same type.

II.METHODOLOGY

1. Identification DC Motor

In this step, the process of identifying DC motor specifications that will be the subject of research is carried out. The DC motor used in the research is a brushless motor with the name and type MG-16B. Figure 1 contains a datasheet that presents information regarding the specifications of the MG-16B DC Motor.

Gear Ratio	Rated Voltage	Part Number	Torque (mNm)			Speed (min⁻¹) (Reference)		Current (mA)			Length (mm)		Weight (g) (Reference)
			Rated	Max.	Starting	No-Load	Rated	No-Load	Rated	Starting	Motor	Gear	
1/30	DC6 V	MG16B-030-AA-00	20	30	> 50	477	380	< 160	< 400	< 1600	35	14	25
	DC12 V	MG16B-030-AB-00						< 80	< 250	< 800			
1/60	DC6 V	MG16B-060-AA-00	40	60	> 100	213	160	< 160	< 400	< 1600	38	17	30
	DC12 V	MG16B-060-AB-00						< 80	< 250	< 800			
1/120	DC6 V	MG16B-120-AA-00	60	90	> 170	127	100	< 160	< 400	< 1600			
	DC12 V	MG16B-120-AB-00						< 80	< 250	< 800			
1/240	DC6 V	MG16B-240-AA-00	120	180	> 350	53	40	< 160	< 400	< 1600	41	20	35
	DC12 V	MG16B-240-AB-00						< 80	< 250	< 800			
1/300	DC6 V	MG16B-300-AA-00	160	240	> 400	45	34	< 160	< 400	< 1600			
	DC12 V	MG16B-300-AB-00						< 80	< 250	< 800			

Figure 1. Datasheet Motor DC MG-16B

2. 2nd Order DC Motor Modeling

The second order DC motor model can be written mathematically as follows [14]:

$$G(s) = \frac{\omega n^2}{s^2 + 2\varsigma \omega n s + \omega n^2} \quad (1)$$

ζ = Damping Ratio (dB)
ωn = Natural Frequencies (rad/s)

Where:

$$\omega n = 2\pi f \quad (2)$$

$\omega n = 2 \times 3{,}14 \times 50 = 314$ rad/s

So the calculations for the MG-16B DC motor are based on the datasheet obtained, namely:

$$G(s) = \frac{\omega n^2}{s^2 + 2\varsigma \omega n s + \omega n^2} \quad (3)$$

$$G(s) = \frac{2\pi f^2}{s^2 + 2\varsigma (2\pi f) s + 2\pi f^2} \quad (4)$$

$$(s) = \frac{2\pi 50^2}{s^2 + 2.30.(2\pi 50) s + 2\pi 50^2} \quad (5)$$

$$G(s) = \frac{98596}{s^2 + 18840 s + 98596 ¿} \quad (6)$$

¿

3. *Linear Quadratic Regulator* (LQR)

Linear Quadratic Regulator (LQR) is one of the methods applied in the domain of modern control theory[15][16]. Analysis of this system involves a state-space based approach. Due to the advantages of this simple approach, this method can also be applied to multi-input multi-output systems. The general state-space equation for the system is expressed as follows:

$$\dot{X} = AX + Bu \quad (7)$$

Basically, the LQR method finds the control signal u that minimizes the performance index J.

$$J = \int \left(X^T Q_x + u^T R_a \right) dt \quad (8)$$

LQR finds the optimal control input law u*. The constraints caused by the matrices Q and R minimize the performance index. The closed loop optimal control law is defined as:

$$u^{¿} = -Kx \quad (9)$$

In this context, K is the optimal feedback gain matrix which functions to reduce the performance index. This gain matrix determines the optimal closed loop root placement to minimize the index. The calculation of the gain feedback matrix K depends on the matrices A, B, Q, and R. The gain feedback matrix K is found by solving the Algebraic Riccati Equation or ARE [17][18]. The positive definite symmetric matrix P is used to find the solution of ARE and is defined as follows:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (10)$$

$$K = AX - BKx = (A - BK)x \quad (11)$$

Substituting (8) and (9) becomes

$$x = AX - BKx = (A - BK)x \quad (12)$$

The LQR configuration block diagram is shown in the image below:

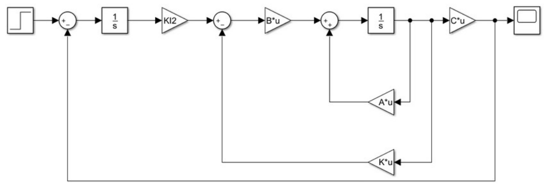


Figure 2. Block Diagram LQR

4. Program Matlab LQR in Motor DC MG-16B

```
% OPTIMASI SISTEM LQR PADA MOTOR DC
clear; clc;
% Model Motor DC
J=46.045 ; b=0.2 ; K=0.5 ; R=0.05 ; L =0.000416 ;
% J = Momeninesia , b = Rasioredam, K= konstanta, R= resistansi, L=
% Induktansi

A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0];

AA = [ A zeros(2,1); -C 0];
BB = [B;0];

% Pole Placement
J = [-3 -4 -5];
K = acker(AA,BB,J)
KI = -K(3);
KK = [K(1) K(2)];

% Matrix LQR
Q = [1 0 0;
   0 1 0;
   0 0 1000];
R = [1] ;

K_lqr = lqr(AA,BB,Q,R)
KI2=-K_lqr(3);
KK2=[K_lqr(1) K_lqr(2)];
```

5. *Linear Quadratic Tracker* (LQT)

LQT is a combination of the conventional state feedback of a linear dynamic system, augmented with an extra feedforward control term[19] . This feedforward control term depends on the reference signal vector, which we represent as r(t). The vector r(t) is expressed in the following form:

$$r(t)=¿ \qquad (13)$$

Where, V_ref is a reference voltage signal that changes with time. The LQT scheme minimizes the squared performance index to produce optimal control decisions which can be formulated in the following equation (Saleem et al., 2018).

$$J=\frac{1}{2}\int_{0}^{T} ¿¿ \qquad (14)$$

In this context, Q and R are weighting matrices for system and control conditions, respectively. This matrix is chosen carefully, namely Q = QT ≥ 0 and R = RT > 0 . Due to the quadratic nature of the cost function, the magnitude of the variability of the system conditions will be proportional to the square of the control signal variation [20]. Therefore, if the variation in the state of the system is large, then the minimization process will converge more quickly.

The optimal affine control decision is evaluated through the mathematical expression shown in (Saleem et al., 2018),

$$d(t)=-Kx(t)+K_{ff}v_{ref}(t) \qquad (15)$$

Where,

$$K=R^{-1}B^{T}P \qquad (16)$$

$$K_{ff}=R^{-1}B^{T}¿ \qquad (17)$$

The gain vector, K, plays a role in shifting the position of the system poles to create an optimal controller. The optimal vector gain depends on the symmetric positive definite matrix, P, shown in equation (14). The matrix P, associated with a given system, can be calculated by solving the Algebraic Riccati Equation, as shown below.

$$A^{T}P+PA-PBR^{-1}B^{T}P+H^{T}QH=0 \qquad (18)$$

The LQT configuration block diagram is shown in the image below:

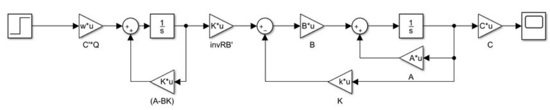


Figure 3. Block Diagram LQT

6. Program Matlab LQT in Motor DC MG-16B

```
% OPTIMASI SISTEM LQR PADA MOTOR DC
clear; clc;
% Model Motor DC
J=0.00000242 ; b=0.1 ; K=0.0167 ; R=0.343 ; L =0.00018 ;
% J = Momeninesia , b =Rasioredam, K=konstanta, R=resistansi, L=Induktansi

A = [-b/J K/J; -K/L -R/L];
B = [0; 1/L];
C = [1 0];
D = 0;
Q=10000;
R=0.001;

%Perhitungan Ricatti
%S=P
[S,o,m,n]=care(A,B,C'*Q*C,R);
k = inv(R)*B'*S
w = C'*Q;
```

```
AB =(A-B*k)';
ABT = AB';
RB=inv(R)*B';
```

7. Block Diagram System

1. Block Diagram of Second Order DC Motor MG-16B

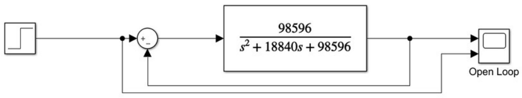


Figure 4. DC Motor Second Order Block Diagram

The first order block diagram of the motor is used to evaluate the original response of the MG-16B DC motor without applying additional methods via Simulink software.
In Figure 4, there is a second order block diagram representing a DC motor, consisting of one input and one output. The input used is a step response signal. The transfer function in the diagram is used to model a second order DC motor. The response results will be visualized through the scope and display to determine the maximum value of the response produced.

2. Block Diagram LQR DC Motor MG-16B

The MG-16B DC Motor LQR block diagram is used to observe the response of the DC motor when the LQR optimization method is applied via Simulink software. The aim is to evaluate the response of a DC motor when the LQR optimization method is implemented.

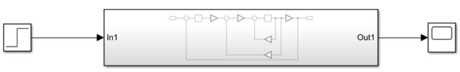


Figure 5. Block Diagram LQR Motor DC MG-16B

3. Block Diagram LQR DC Motor MG-16B with Noise

The MG-16B DC Motor LQR block diagram with added noise is used to observe the response of the DC motor when the LQR optimization method is applied and noise is integrated into the system via Simulink software. The aim is to evaluate the response of a DC motor when the LQR optimization method and noise elements are applied in the system.

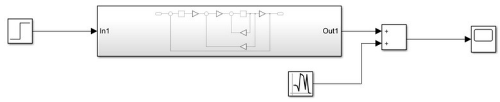
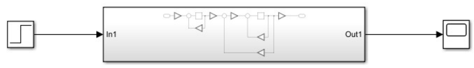


Figure 6. Diagram Blok LQR Motor DC MG-16B With Noise

4. Blok Diagram LQT Motor DC MG-16B



Gambar 7. Block Diagram LQT Motor DC MG-16B

5. LQT Block Diagram of MG-16B DC Motor with Noise

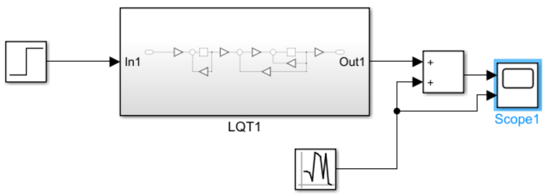


Figure 8. Block Diagram LQT Motor DC MG-16B with *Noise*

III. RESULT & DISCUSION

In this section, the response of the MG-16B DC motor to the first order mathematical model will be examined, as well as the effect of applying the LQR and LQT methods, both with and without additional noise elements. Response data was obtained through simulation using Simulink Matlab software.

1. Second Order Response Results of MG-16B DC Motor

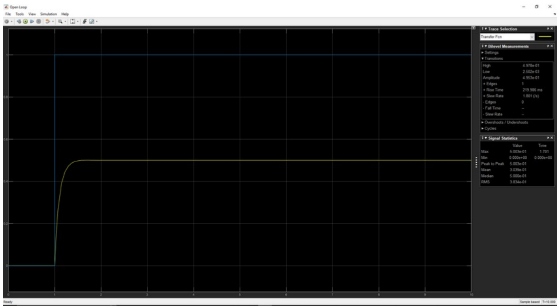


Figure 9. MG-16B DC Motor Response Results in Second Order Mathematical Modeling

In the second order model response output, it can be seen that the motor response graph has a significant value from the desired set point. The yellow line depicts the motor response, while the blue line reflects the desired set point, namely 0.5. The motor response only reaches around 219 with high ripple during the response phase. The observed MG-16B DC motor shows linear characteristics, indicated by a response curve that is free from fluctuations. The motor response reaches steady state conditions at around ±0.27 seconds.

2. MG-16B DC Motor Response Results Using the LQR Method

Figure 10. MG-16B DC Motor Response Results using the LQR Method

In Figure 10, you can see the step response display of the MG-16B DC motor using the LQR method without any noise. It can be observed that the step response of the MG-16B DC motor using LQR reaches an amplitude of about 1.033, which can be rounded to 1, indicating the achievement of the desired setpoint. This response has a minimal rise time, around 3.940 seconds, and has very small overshoots and undershoots, around 0%.

3. MG-16B DC Motor Response Results Using the LQR Method with Noise

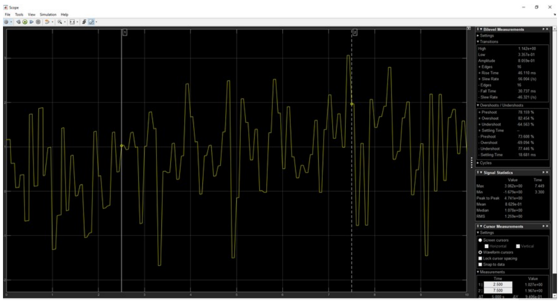


Figure 11. Response Results of the MG-16B DC Motor using the LQR Method When Noise is Given

In Figure 11, you can see the step response display of the MG-16B DC motor using the LQR method which is affected by noise. The step response of the MG-16B DC motor using LQR in this condition has a fluctuating graph due to interference from noise. The system amplitude reached approximately 8.059, which means the system has not yet reached the desired setpoint. The response rise time was quite maximum, around 52,720 milliseconds, and the system experienced an overshoot of 82.454% and an undershoot of -64.563%.

4. MG-16B DC Motor Response Results Using the LQT Method

Figure 12. MG-16B DC Motor Response Results using the LQT Method

In the modeling response graph shown in Figure 12, it can be seen that the motor response of the MG-16B DC Motor using the LQT method reaches a value that corresponds to the desired set point, namely 0.5. The response time to steady state conditions is very fast, only around 8,364 microseconds. However, the resulting response has an overshoot of 3.646% and an undershoot of 0.477%.

5. Comparison Results of MG-16B DC Motor Response Using LQT Method and Without Method

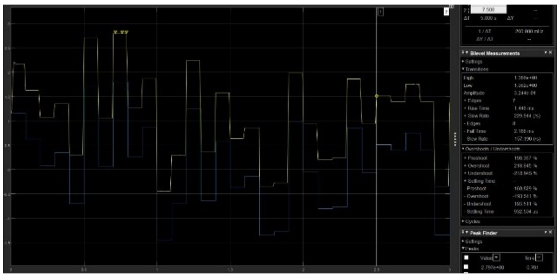


Figure 13. Response results of the MG-16B DC motor using the LQT method when noise is applied

From the data shown in Figure 13, it can be seen that the yellow signal which is the response of the MG-16B DC motor using the LQT method changes shape after being given noise. The shape of this signal experiences many variations (ripple) and resembles the noise signal pattern added previously. The resulting signal is no longer linear and is far from reaching a stable condition at a predetermined set point.

IV.CONCLUSION

Based on the practical results with the MG-16B DC motor, it was found that the use of the LQR method produced a response in accordance with the desired set point value without any overshoot or undershoot. The time required to reach steady state conditions is also shorter, around ±1.2 times faster than without using this method. When a DC motor is operated without the LQR method, the motor response is far from the desired set point value and requires quite a long time to reach steady state conditions.
However, when the LQR method is used with additional noise in the system, the motor response follows the pattern of the added noise signal, so the resulting response is not linear. When using the LQT method, the time to reach steady state conditions is very fast, around 8,364 microseconds. Even so, the resulting response still experienced an overshoot of 3.646% and an undershoot of 0.477%. The results of the motor response using the LQT method when noise is applied also show a response pattern that follows the noise signal in the system, which makes it non-linear.
Overall, the LQT method is considered better than the LQR method because it produces a faster response than the LQR method to reach the set point on the MG-16B DC motor.

V. REFERENCE

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