

Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) control systems on M66 Series DC motors.

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ABSTRACT

Some players in the industry seek to build or improve systems in their industry to produce superior products compared to their competitors. Recently, attention to optimal control problems has increased due to the increasing demand for high-performance systems. The concept of control system optimization involves the selection and design of key performance indicators that will guide the creation of an optimal control system within existing physical constraints. When designing an optimal control system, the goal is to find decision rules that will lead to control actions that minimize deviations from the desired behavior. LQR is one of the optimal control methods used in state space based systems. The LQR controller has two parameters, namely the weight matrices Q and R, which must be determined to achieve optimal control actions. LQT is an optimal control method that aims to minimize the objective function (Performance Index) and regulate the system (Plant) so that it can follow the desired reference. The step response results of the M66 Series DC motor using LQR show that the amplitude reaches around 0.799, which can be considered as 1 and reaching the setpoint. On the other hand, the step response of the LQT system produces a higher amplitude, around 0.99, which also reaches the setpoint. In addition, the LQT response has a faster rise time compared to the LQR response, around 1,509 milliseconds, and experiences an overshoot of around 1.05% and an undershoot of around 1.05%. Overall, it can be concluded that the M66 Series DC motor using LQR produces more optimal results compared to the 1st order M66 Series DC motor. This can be seen from the system's ability to use LQR to achieve setpoint, stable graphics, fast rise time, and overshoot value. and lower undershoot.

Key Word: Control, LQR, LQT, DC Motor, Noise, Optimal

I. INTRODUCTION

Rapid technological advances have a significant impact on various aspects of life today. It cannot be denied that the demand for technology has increased significantly compared to several decades ago. The impact can be seen in global competition, especially in the industrial sector. Many industry players have attempted to build or improve systems within their industry to produce products that are superior to those of their competitors. Optimal control has become an increasingly important issue recently due to increasing demands on high-performance systems [1][2]. The concept of control system optimization

involves selecting and designing key performance indicators that will guide the creation of an optimal control system within existing physical constraints [3]. When designing an optimal control system, the goal is to find decision rules that will produce a control system that can minimize deviations from desired behavior [4].

The goal of system optimization is to determine appropriate control signals so that the system output can maintain or follow the given reference, while also minimizing or maximizing certain performance indices [5]. Optimal control can be applied in two main cases, namely the regulator case (known as Linear Quadratic

Regulator) and the tracking case (known as Linear Quadratic Tracking) [6][7].

In this report, the author discusses the LQR (Linear Quadratic Regulator) and LQT (Linear Quadratic Tracking) methods related to the PPNS Automation Engineering course entitled "System Optimization"[8].

II. METHODOLOGY

2.1 LQR (Linear Quadratic Regulator)

LQR is an optimal control method for state space systems [9]. The LQR controller has two parameters, namely the weight matrices Q and R , which need to be determined in order to be able to provide appropriate optimal control actions [10]. Examples of LQR implementation include regulating the speed of induction motors, frequency control in power plants with generators, and quadcopter drones. When using LQR, the system will maintain a state at zero relative to the set point to maintain system stability even when experiencing noise/interference [11].

2.2 LQT (Linear Quadratic Tracking)

LQT is an optimal control method which aims to minimize the objective function (Performance Index) and regulate the system (Plant) so that it can follow the desired reference [12]. The system actuator controller is the result of a passive controller. This actuator uses the voltage value generated by LQT based on the model and setpoint that has been set [13]. LQT was chosen because this method uses an analytical approach and is often used for tracking problems [14].

This research becomes interesting to explore when LQT is applied to PV tracking systems in passive mode. The aim of this research is to control the Maxon EC60FLAT

DC motor in such a way using analytical calculations. Thus, this paper will produce and analyze the implementation of LQT on a Maxon EC60FLAT DC motor with one axis that only uses passive control[15]. Next, this research will carry out a simulation analysis with variations in several parameters, such as the moment of inertia (J), on the mathematical model of a DC motor.

2.3 DC Motor

DC Electric Motor, or known as DC Motor, is a device that converts electrical energy into kinetic energy or movement [16]. This DC motor is also often referred to as a Direct Current Motor. As the name suggests, a DC motor consists of two terminals and requires direct current or DC (Direct Current) voltage to drive it [17]. DC Electric Motors are generally used in electronic and electrical devices that use DC electrical power sources, such as cellphone vibrators, DC fans, and DC electric drills [18]. The construction of this DC motor can be seen in Figure 1.

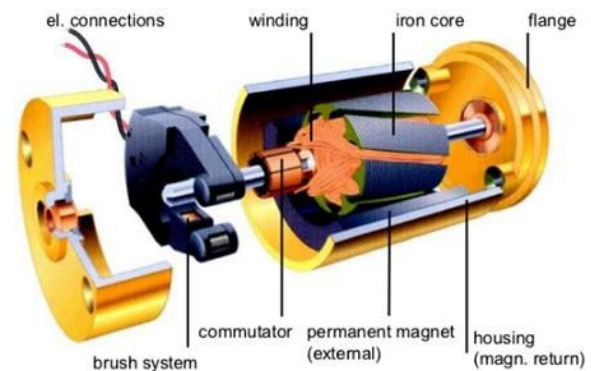


Figure 1. Construction DC motor

2.4 Matlab

MATLAB is a programming platform that uses a matrix language. Therefore, it is often used to analyze data, create algorithms, and create models and applications[19]. The MATLAB software screen displays as shown in Figure 2.

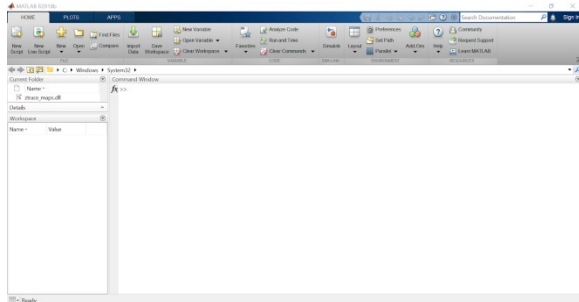


Figure 2. MATLAB

In order to carry out a simulation to observe the response produced by a DC motor, the author utilizes the Simulink feature in MATLAB. Simulink is a component of MATLAB which functions as a graphical programming tool.

The main function of Simulink is to create dynamic system simulations [20]. This simulation process is carried out using a functional diagram consisting of connected blocks, each of which has an equivalent specific function. Simulink is used as a dynamic system modeling, simulation and analysis tool through a graphical user interface. Simulink consists of various groups of tools that can be used to analyze systems, both linear and non-linear.

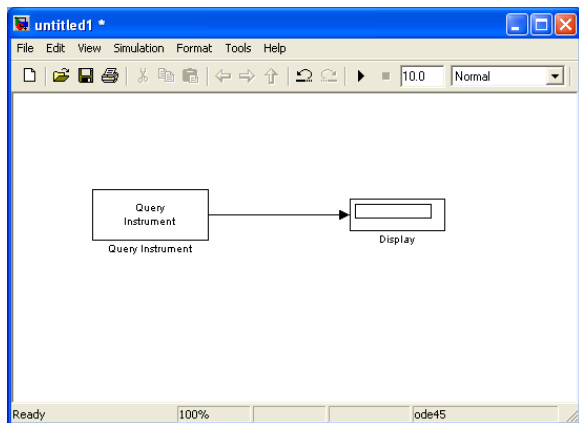


Figure 3. Simulink MATLAB

2.5 Noise

Noise or disturbance is a signal that has a tendency to influence the output value of a

system. Disturbances that arise from within the system are referred to as internal disturbances, while disturbances that come from outside the system are referred to as external disturbances. This disturbance will of course cause the output value to not be as desired [10].

2.6 DC Motor Identification

DC motors, also known as direct current motors, are a type of electric motor that is capable of converting direct current electrical energy into mechanical energy [3]. Its operating principle is based on the interaction between two magnetic fluxes referred to as a field coil and an armature coil. The result is the generation of energy in the form of rotation. A DC motor consists of two main components, namely the stator (non-rotating part) which contains the field coil, and the rotor (rotating part) which contains the anchor coil [2]. In this experiment, the motor used was the M66 Series DC Motor. The following are the specifications of the M66 Series DC Motor.

Specification: Special models

Special models:	M66CE-12/ DB24	M66CE-24 /DB12
Nominal Motor Voltage	12 Vdc	24 Vdc
Nominal Brake Voltage	24 Vdc	12 Vdc
Specification		
Applied motor Voltage (Vdc)	12	30
Maximum Output Power (Watts)	15	30
No-load speed (rpm)	2,700	2,900
Speed @ rated torque (rpm)	1,800	2,300
Rated Torque (Ncm)	8	12
Peak Torque (Ncm)	25	36
Max. No load current (milli Amps)	120	65
Rotor Inertia (Kgcm ²)	0.214	0.214
Mechanical time constant (milli secs)	24.5	17
Torque Constant (Ncm / A)	4.1	9.8
Voltage Constant (V / 1000 rpm)	4.27	10.3
Rotor Resistance (Ohms)	1.9	7.8
Rotor inductance (mH)	1.0	5.0
Commutation	copper -graphite	
Bearings	pre-loaded ball	
Maximum radial load	100 N, 12 mm from bearing face	
Maximum axial load	15 N	
Ambient operating temperature range	-10 to +60 °C	
Brake specification		
Nominal Brake Voltage (Vdc)	24	12
Operating Current (Amps)	0.22	0.50
Min. Pull-in voltage (Vdc)	18	10
Max. Drop-out voltage (Vdc)	8	6
Holding Torque, de-energised (Ncm)	10	10

Figure 4. M66 Series DC Motor Specifications

- Moment of inertia(J): 0.214 kg.m²/s²

- Damping of mechanical systems(B): 0,1 Nms
- Motor Constant(K): 0.32 Nm/A
- Resistance(R): 1.9 ohm
- Induction(L): 0.001 H

J = 0.214 ; b= 0.1 ; K= 0.32 ; R= 1.9 ; L = 0.001 ;

% J = Momenesia , b = Rasioedam, K= konstanta, R= resistansi, L=

From the data sheet, we can determine the mathematical model of the IG-22GM DC motor system as a first order system. A first order system is a system in which one change occurs [10]. The following are details of the first order system modeling.

% Induktansi

A = [-b/J K/J; -K/L -R/L];

B = [0; 1/L];

C = [1 0]

Equation of order transfer function 1 :

AA = [A zeros(2,1); -C 0];

$$G(s) = \frac{K}{\tau s + 1} \quad (1)$$

BB = [B;0];

Based on the IG-22GM DC motor datasheet, the 1st order equation is obtained:

% Pole Placement

J = [-3 -4 -5];

Where = K. i so that the 1st order equation for a dc motor:

K = acker(AA,BB,J);

$$K = \frac{\tau}{i} = \frac{0,08}{0,25} = 0,32 \text{ nm} \quad (2)$$

KI = -K(3);

KK = [K(1) K(2)];

$$G(s) = \frac{0,32}{0,08s + 1} \quad (3)$$

% Matrix LQR

Q = [1 0 0;

0 1 0;

0 0 1000];

2.7 Program Script Matlab

2.7.1 Program Script Matlab LQR

% OPTIMASI SISTEM LQR PADA MOTOR DC m66 series

R = [1] ;

K_lqr = lqr(AA,BB,Q,R)

clear;

KI2 = -K_lqr(3);

clc;

KK2 = [K_lqr(1) K_lqr(2)];

% Model Motor DC

2.7.2 Program Script Matlab LQT

```
% OPTIMASI SISTEM LQR PADA MOTOR DC  
m66 series
```

```
clear;
```

```
clc;
```

```
% Model Motor DC
```

```
J = 0.214 ; b= 0.1 ; K= 0.32 ; R= 1.9 ; L =  
0.001 ;
```

```
% J = Momeninertia , b = Rasioedam, K=  
konstanta, R= resistansi, L=Induktansi
```

```
A = [-b/J K/J; -K/L -R/L];
```

```
B = [0; 1/L];
```

```
C = [1 0]
```

```
Q=10; R=0.0000000001;  
%0.000000000000001
```

```
W=C'*Q; %
```

```
[S,o,m,n]=care(A,B,C'*Q*C,R) %m=v(t) %S=P  
K=inv(R)*B'*S %feedback Gain
```

```
ACL=(A-B*K)'
```

```
L=inv(R)*B' %model following gain
```







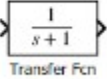



2.8 LQR dan LQT

MATLAB is a programming platform specifically designed for numerical processing purposes (Marwan E Endy, 2019). According to MathWorks, the main developer of MATLAB, this platform uses a matrix-based language and is usually used

for data analysis, algorithm development, and modeling and application creation. In this experiment, I used MATLAB software, including one of its features called Simulink. Simulink is a part of MATLAB that functions as a graphical programming tool for creating dynamic system simulations. The simulation process is carried out using a functional diagram consisting of blocks that are connected to each other according to their respective functions.

2.8.1 Table of Component

Table 1. Matlab Component

 Add	 Gain
 Step	 Integrator
 Random Number	 Scope
 Transfer Fcn	 Sum
 Out1	 In1

2.8.2 DC Motor Circuit M66 Series

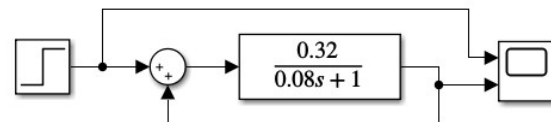


Figure 5. DC Motor Circuit M66 Series

2.8.3 LQR

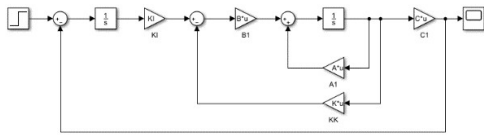


Figure 6. LQR

2.8.4 Subsystem LQR Without Noise

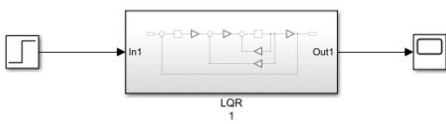


Figure 7. Subsystem LQR Without Noise

2.8.5 Subsystem LQR With Noise

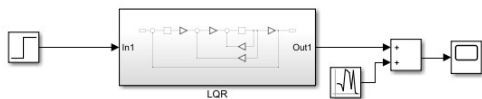


Figure 8. Subsystem LQR With Noise

2.8.6 LQT

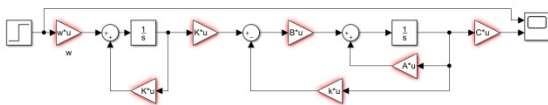


Figure 9. LQT

2.8.7 Subsystem LQT With Noise

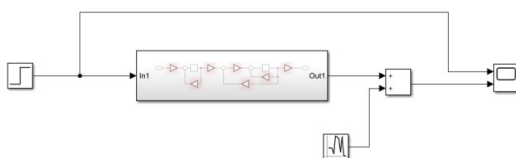


Figure 10. Subsystem LQT With Noise

2.8.8 Subsystem LQT Without Noise

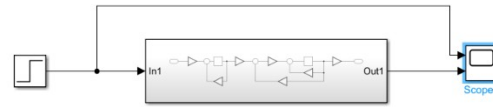


Figure 11. Subsystem LQT Without Noise

III. RESULT & DISCUSSION

3.1. Simulation Result

3.1.1 M66 Series DC Motor Simulation Results

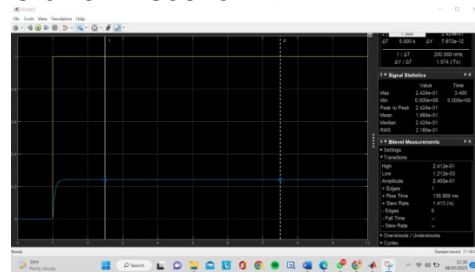


Figure 12. Step Response

Figure 12 displays the step response of the M66 Series DC motor in first order LQR control without any disturbance. The response graph for this step shows stable results with an amplitude of around 0.24 (which does not reach the set point), a rise time of around 0.1359 seconds, and an overshoot of around 0.501% and an undershoot of around 0.472%.

3.1.2 Simulation LQR Without Noise



Figure 13. Step Response LQR Without Noise

Figure 13 shows the step response display of a noise-free M66 series DC motor. It can be seen that the output step response of

the M66 series LQR DC motor reaches a magnitude of 0.799 which can be rounded up to 1 until it almost reaches the set point. It has a decent maximum rise time of 1,509 ms and fairly low overshoot and overshoot of 1.05%.

3.1.3 Simulation LQR With Noise

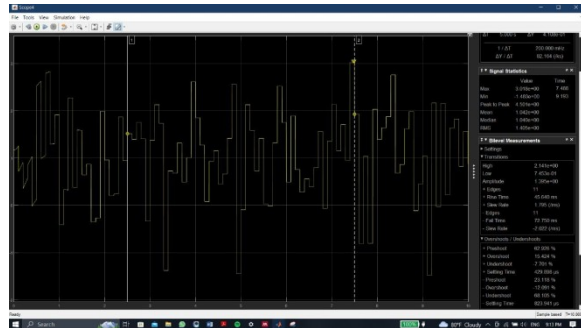


Figure 14 displays the step response of the M66 Series DC motor in LQR control in the presence of noise. It can be seen that the output stroke response of the M66 Series DC motor controlled by LQR experiences fluctuations in the graph due to existing disturbances. The system reaches an amplitude of around 1.395, so it has not yet reached setpoint. This response has a rise time that reaches a maximum value of around 45,640 milliseconds and experiences an overshoot of around 15,425%, and an undershoot of around -12,091%.

Table 2 Simulation data system LQR

No	Model System	Amplitude	Rise Time (Ms)	Overshoot %	Undershoot %
1	LQR Without Noise	0.799	1.509	1.05	1.05
2	LQR Noise	1.395	45.640	15.425	-12.09

3.1.4 Simulation LQT Without Noise



Figure 15. Step Response LQT Without Noise

Figure 15 shows the step response of the M66 Series DC motor without any interference (noise). It can be seen that the output step response reaches an amplitude of about 0.99, which can be rounded to 1, thereby reaching the setpoint. This response has a fairly maximum rise time of around 1.646 milliseconds, and experiences an overshoot of around 1.531%, and a relatively small undershoot of around 1.18%.

3.1.5 Simulation LQT with Noise

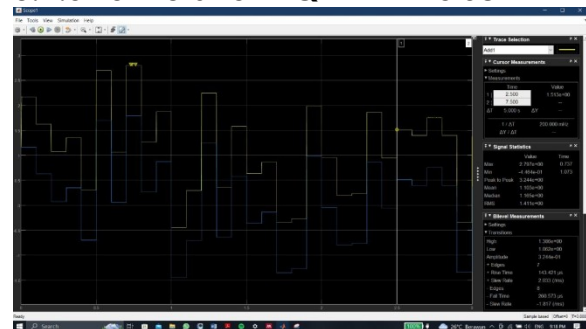


Figure 16 displays the step response of the M66 Series DC motor which is controlled using LQR in the presence of noise. It can be seen that the output step response of the M66 DC motor which uses LQR experiences fluctuations in the graph due to the disturbance provided. The system reaches an amplitude of around 0.324, so it has not yet reached setpoint. This response has a rise time that reaches a maximum value of around 143.421 milliseconds and experiences an overshoot of around -

193.474%, and an undershoot of around -193.474%.

Tabel 2. Simulation Data System LQT

n o	Model sistem	Amplitudo	Rise time (ms)	Overshoot (%)	Undershoot (%)
1	LQT Without Noise	0.99	1.646	1.531	1.18
2	LQT Noise	0.324	143.421	-193.474	193.474

IV. CONCLUSION

- To obtain a mathematical model of a 1st order DC motor and the variables needed to control LQR, a DC motor datasheet is needed which provides information about the moment of inertia, motor constant, damping ratio, resistance and inductance. By carrying out 1st order mathematical modeling calculations, we can find the transfer function $G(s) = \frac{0,029}{0,231s+1}$. Next, by running the Matlab LQR script, we can obtain the values of variables such as A, B, C, K_lqr, and others that will appear in the workspace.
- The step response results of the 1st order M66 Series DC motor show a stable response with an amplitude of around 0.24, which can be interpreted as reaching a setpoint of around 1, with a rise time of around 0.1359 seconds. Apart from that, this system experienced an overshoot of around 0.501% and an undershoot of around 0.472%.

On the other hand, the step response of the M66 Series DC motor controlled with LQR reaches an amplitude of about 0.799, which can be regarded as reaching a setpoint of about 1.

- The step response from the LQT system produces more optimal results compared to the response from LQR. This response reaches an amplitude of about 0.99, which can be considered to reach a setpoint of about 1. Moreover, its rise time is more optimal compared to the rise time of the step response of the uninterrupted LQR, which is about 1,509 milliseconds. This response also has an overshoot of about 1.05% and an undershoot of about 1.05%.
- It can be seen and concluded from the step response results of both systems that the use of LQR on the M66 Series DC motor produces more optimal results compared to the 1st order M66 Series DC motor. By using LQR, the step response of the M66 Series DC motor is able to reach setpoint, indicating the graph is stable, has a faster rise time, and has overshoot and undershoot with smaller values.

V. REFERENCE

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